Dynamic Price Competition and Evolutionary Behavior with Search: Online Appendix*

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This online appendix contains three sections. In Section A we illustrate how the model can be extended to N firms and discuss the assumption regarding α_i being constant across time. In Section B we illustrate how the residual maximizer can either decrease or increase as search becomes less frequent with respect to first order stochastic dominance. In Section C we discuss the case of forward looking firms.

Online Appendix A

A.1 Extension to N Firms

The results Sections 2 and 3 can be generalized to the N firm case. The proofs of the Propositions and Theorems are built upon the judo prices and residual maximizers. The residual maximizers are independent of the number of firms and are therefore unchanged with N firms. Hence, the results can be generalized to the N firm case by augmenting the definition of the judo price. For simplicity and expositional convenience, we consider the clearinghouse model (Salop and Stiglitz, 1977), though the results also generalize to sequential search.

Let $p^t = (p_1^t, \ldots, p_n^t)$, where p_{-i}^t has the typical interpretation and let α_i denote the mass of consumers that observe firm i's price with $\sum_{j=1}^N \alpha_j = 1$. We first construct the demand facing each firm as a function of the price vector p^t and the distribution of consumers' thresholds x^t. If a firm i's price is lower than all of the other firms, then it will serve those consumers that initially observe p_i^t as well as all of the consumers that observe other prices and search. If firm i and $b-1$ other firms have the lowest price then firm i will serve the consumers that initially observe p_i^t and do not search as well as $1/b$ of all of the consumers that search. If firm i does not have the lowest price, then it will serve only the consumers that initially observe p_i^t and do not search. Let $I(p^t) = \{i : p_i^t = \min_j p_j^t\}$

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and $J(p^t) = \{i : p_i^t > \min_j p_j^t\}$. The demand facing each firm i is

$$D_i\left(p^t, x^t\right) = \begin{cases} \alpha_i + \sum_{j \neq i} \alpha_j \bar{\varphi}(p_j^t, x^t) & \text{if } p_i^t < \min_{j \neq i} p_j^t \\ \alpha_i + \frac{1}{|I(p^t)|} \sum_{j \in J(p^t)} \alpha_j \bar{\varphi}(p_j^t, x^t) & \text{if } p_i^t = \min_{j \neq i} p_j^t \\ \alpha_i(1 - \bar{\varphi}(p_i^t, x^t)) & \text{if } p_i^t > \min_{j \neq i} p_j^t. \end{cases}$$

The front-side and residual profits are now

$$\pi_i^F(p, x) = p_i \left(\alpha_i + \sum_{j \neq i} \alpha_j \bar{\varphi}(p_j, x) \right)$$
$$\pi_i^R(\xi, x) = \xi \alpha_i (1 - \bar{\varphi}(\xi, x)).$$

The best response correspondence with N firms is nearly identical to that when there are only two firms with an appropriately revised Judo price. Rather than a firm's preference between monopolizing its residual demand and undercutting being determined entirely by the price that it must set in order to undercut, this preference is determined by all the prices set by the competing firms, not just the lowest price. The reason is that as some firms charge higher prices, more of the consumers that observe their prices will search, thereby creating a higher incentive for the remaining firms to set the lowest price in order to attract those searching consumers.

We generalize the notion of the judo price to the judo correspondence, defined analogously to the judo price with two firms. For each pair of firms i, j, let p_{-ij} denote the vector of prices for the firms that are neither i nor j. For any vector z, let $\underline{\underline{p}}(z) = \min z$ denote the smallest component of z. Define the conditional judo price for firm i relative to firm j given the vector p_{-ij} as

$$p_{ij}^*(p_{-ij}, x) = \max \Big\{ g_\omega \in G : \min \left\{ g_{\omega-1}, \underline{\underline{p}}(p_{-ij}) \right\} \Big(\alpha_i + \alpha_j \bar{\varphi}(g_\omega, x) + \sum\nolimits_{h \neq i,j} \alpha_h \bar{\varphi}(p_h, x) \Big) < \max_{p_i} \alpha_i p_i (1 - \bar{\varphi}(p_i, x)) \Big\}.$$

If $p_{ij}^*(p_{-ij}, x) = 1$ and

$$\underline{\underline{p}}(p_{-ij}) \left(\alpha_i + \alpha_j \bar{\varphi}(1, x) + \sum\nolimits_{h \neq i,j} \alpha_h \bar{\varphi}(p_h, x) \right) < \max_{p_i} \alpha_i p_i (1 - \bar{\varphi}(p_i, x)),$$

then set $p_{ij}^*(p_{-ij}, x) = \infty$. Given these conditional judo prices, define firm i's judo correspondence to be

$$P_i^*(x) = \bigcup\nolimits_{j \neq i} \bigcup\nolimits_{p_{-ij} \in G} (p_{ij}^*(p_{-ij}, x), p_{-ij}).$$

Note that one immediate difference between the judo price for a duopoly and the conditional judo price is that the conditional judo price may be strictly less than the monopoly price while some of the other prices are below the search cost c, meaning that the firm would maximize its profits by choosing a price below the search cost. The reason is that firms pricing above the search cost induce some amount of search from consumers, and so a firm may wish to undercut other firms that have prices below c in order to attract the searching consumers. This was not possible with only two firms, as if the single other firm has a price below the cost of search, then there are no searching consumers to attract and thus there is no incentive for a firm to undercut that price.

Given this Judo correspondence, under the conditions that satisfy Proposition 1 in the main text, the best response correspondence for N firms is qualitatively similar to the two firm case (each firm will either undercut or monopolize its residual demand). Formally:

$$R_i(p_{-i}, x) = \begin{cases} \{g_{\omega-1}\} & \text{if} \quad p_{-i} \in \text{epi}_S P_i^*(x) \\ \{g_{\omega-1}\} \cup \tilde{P}(x) & \text{if} \quad p_{-i} \in P_i^*(x) \\ \tilde{P}(x) & \text{if} \quad p_{-i} \in \text{hyp}_S P_i^*(x), \end{cases}$$

where $\text{epi}_S P_i^*(x)$ and $\text{hyp}_S P_i^*(x)$ denote the respective strict epigraph and strict hypograph (subgraph) of $P_i^*(x)$. The critical judo price is now $p^*(x) = \max_i \{\inf \text{epi}_S P_i^*(x)\}$ and the subsequent proofs follow analogous arguments to the two firm case.

A.2 Assumption on the Observed Price α_i

The main text assumes that α_i is constant across time, so the firm that a consumer observes is independent of their previous purchase. Though this assumption has no bearing on the results (in the myopic firm specification), it can alter the interpretation of Proposition 4. Before discussing the alternative implication, we first briefly illustrate with an example that this assumption is reasonable in some markets. Consider the retail gasoline market and suppose that there is a road with two gas stations. At one end of the road is the consumer's home and at the other end is the consumer's place of work. The gas stations are at intermediate locations. If a consumer is traveling from home to work, that consumer observes a different gas station's price than if the consumer is traveling from work to home.

Suppose now that α_i is determined by the distribution of purchases, so at each time, α_i is determined by i's previous sales. This would simply move the judo price at each instant, but none of the proofs of the propositions rely on a constant α_i. What matters is the value of α_i at the time that firm i is called to move according to its Poisson process. The best response correspondence of Proposition

1 still governs the pricing decision. Under Proposition 3, the firm with the largest 'installed base' has the stronger incentive to relent (the higher judo price). With a constant α_i and in the context of retail gasoline markets, we can think of the firms with the larger α_i as the 'major brands' and the others as the 'independent brands.' The empirical literature is consistent with the results of Proposition 3: major brands are the first to relent in retail gasoline markets characterized by Edgeworth cycles (Noel, 2007; Atkinson, 2009; Isakower and Wang, 2014).

In the forward-looking case (See Appendix C.2 below), this assumption has a more significant impact on the results. If α_i is determined by previous sales, then there is a greater incentive to boost current sales as this will induce an increased subsequent residual demand. Hence, there is a greater incentive to undercut. However, the judo price is still well defined, so there is still an eventual incentive to relent when price is above marginal cost and the cycles can persist as described.

Online Appendix B

B.1 First Order Stochastic Dominance and Residual Maximizers

Suppose that (i) φ is defined such that if all consumers have a single threshold τ_k, then the residual maximizer is increasing in that τ_k, and (ii) σ has a very large probability of taking on values in the range $(\tilde{P}(x) - \tau_{k+1}, \tilde{P}(x) - \tau_{k-1})$ and a very small probability of taking on other values. Consider two distributions x and x', where under both x and x', half of the consumers have τ_L. Under x, the remaining half of the consumers have threshold τ_k. Under x', the remaining half of the consumers threshold τ_{k-1}.

Thus, under x, there are two candidate residual maximizers: a low maximizer that attempts to serve both types of consumers and a high maximizer, which induces a very high probability that the τ_k consumers search and maximizes the revenue from the τ_L consumers. Now consider x'. At the price $\tilde{P}(x')$, the τ_{k-1} consumers search with near certainty. As such, the low maximizer must be smaller under x', while the high maximizer is the same regardless of the distribution. This lower maximizer may yield a smaller profit under x' than $\tilde{P}(x)$ yields under x. If the difference is large enough, then the residual maximizer under x' would be the high maximizer. Hence, with consumers searching more, the residual maximizer is higher.

B.2 Uniformly Distributed Noise

To show that the upper bound of the cycle is higher under x than x' when x first order stochastically dominates x', it is sufficient that the smallest residual maximizer is under x is greater than the largest residual maximizer under x'. For simplicity, we approximate the model by assuming continuity in prices. Hence, it is sufficient to show that $\frac{\partial \pi_i^R(\xi,x)}{\partial \xi} > \frac{\partial \pi_i^R(\xi,x')}{\partial \xi}$ for all ξ. Note that

$$\frac{\partial \pi_i^R(\xi,x)}{\partial \xi} = (1-\bar{\varphi}(\xi,x)) - \xi \frac{\partial \bar{\varphi}(\xi,x)}{\partial \xi},$$

where $\frac{\partial \bar{\varphi}(\xi,x)}{\partial \xi} = \frac{1}{\text{range}\,\text{supp}\,\varphi} = z$ for some constant $z > 0$. Hence, the upper bound of the cycle is larger under x than x' if

$$(1-\bar{\varphi}(\xi,x)) - \xi z \geq (1-\bar{\varphi}(\xi,x')) - \xi z,$$

which simplifies to

$$\bar{\varphi}(\xi,x) - \bar{\varphi}(\xi,x') \leq 0.$$

This expression holds by Lemma 1 as x first order stochastically dominates x'. Thus, when the noise is uniformly distributed, $\inf \tilde{P}(x) \geq \sup \tilde{P}(x')$ and strictly so when interior. For a sufficiently fine grid, the result extends to a finite grid G.

B.3 A General Result on the Residual Maximizer

There are certain conditions under which we can guarantee monotonicity of the residual maximizer with respect to first order stochastic dominance.

Proposition B1. *If $\bar{\varphi}(\xi,x)$ is submodular in (ξ,x), then $\tilde{P}(x)$ is monotonic in x with respect to first order stochastic dominance.*

Proof. Consider two distributions x and x', where x first order stochastically dominates x'. By Lemma 1, $\bar{\varphi}(\xi,x) \leq \bar{\varphi}(\xi,x')$. Recall that residual profits are

$$\pi_R(\xi,x) = \xi\alpha_i(1-\bar{\varphi}(\xi,x)).$$

The submodularity of $\bar{\varphi}(\xi,x)$ implies that $\pi_R(\xi,x)$ is supermodular. Hence, $\tilde{P}(x)$ exhibits monotone comparative statics on x (with respect to first order stochastic dominance). □

Many distributions satisfy this property on the economically relevant region (e.g., the log-concave unimodal distributions).

Online Appendix C

C.1 Discussion of Myopic Firms

A potential concern is that the results are driven by the choice to model firms as myopic payoff maximizers rather than forward looking. However, this assumption can be motivated by the primitive assumption that firms do not have knowledge of the process by which consumer behavior changes over time. Indeed, there is no assumption (or need to assume) that consumers understand this process, and so even if the managers of the firms are themselves consumers, this need not impart any such knowledge. Despite not understanding the process by which consumers update their strategies, it is possible for firms to observe (or at least estimate) the current state, e.g., via market research, consumer surveys, or focus groups. Given sufficient data, the firms can accurately predict the current state, though no amount of data is sufficient to identify the consumer process. Without an understanding of the consumer dynamic, it is not possible for firms to form meaningful expectations as to how the distribution of consumers' price thresholds will change in response to the prices that are set. For example, firms may recognize the possibility that the consumers act as they do in the model, while at the same time anticipating that consumers search more (lower their thresholds) whenever prices are high. The latter belief could be derived from the anticipation that consumers strategically search more in response to high prices so as to induce firms to lower their prices in the future.

In addition to uncertainty regarding the nature of the consumer dynamic, there is an additional lack of information regarding the rate at which the consumers change their strategies. With so little information, it is unreasonable to expect that firms anticipate the consumers' reactions to their prices. Any prior that the firms have over the possible consumer dynamics ought to be uninformative, which would assign an equal probability to any response and its opposite, leading to a belief that the consumer dynamic will remain unchanged in expectation. While the dynamic may be fixed in expectation, the realization of the distribution of consumers' price thresholds will almost certainly change before either firm gets another opportunity to change its price. As such, forward looking optimization would require maximizing the net present expected value of the profits, which requires further anticipation of each firm's response in all possible states and all possible posterior beliefs about the consumer dynamic in all future periods. Given the staggering variance of predictions about future behavior and the immense computational burden, it seems reasonable to assume that firms either are unable to determine the optimal pricing strategy to maximize their stream of profits and instead opt to take the more simple approach of maximizing short run profits. Empirical evidence suggests such managerial costs can be high (Zbaracki et al., 2004) and that behavioral-based substitutes are often utilized (Blinder et al., 1998).

C.2 Intuition for Extension to Forward Looking Firms

The primary benefit of the myopic firm assumption is the precision it grants in identifying the short-run dynamics. Here, we informally illustrate that the general patterns identified in the main results can (with some minor additional assumptions) carry over to the forward-looking case, though much of the precision is lost. Hence, the main results are primarily driven by the demand side of the market rather than the myopic firm assumption. A complete formal characterization of the forward looking case is left for future work.

Let λ_i denote firm i's Poisson parameter and $\beta > 0$ the common discount rate. Throughout, profits refer to expected profits and we will assume that $g_m - g_{m-1} = \delta$ for all m (prices on the grid are evenly spaced) and that δ is sufficiently small with respect to the search cost c so that for any $p = (g_m, g_{m-1})$ and $m \geq 1$, $c > c^*(p)$. That is, search has a negative expected utility when the firms' prices are adjacent.

C.2.1 The Best Response Correspondence

For simplicity, we apply a mean field approach to the evolution of x^t so we can focus on the firms. Define $\Lambda = \lambda_1 + \lambda_2$ and $T \sim \text{Exp}(\Lambda)$. Let $\pi_i(p_i, p_{-i}, x)$ denote firm i's instantaneous profits and

$$V_i(p_i, p_{-i}, x) = \mathbb{E}\left(\int_0^T e^{-\beta t}\pi_i(p_i, p_{-i}, x)dt + e^{-\beta T}W_i(x^T)\right)$$

as firm i's value function, where

$$\begin{aligned} W_i(x) &= \frac{\lambda_i}{\Lambda}\max_{\xi\in G} V_i(\xi, p_{-i}, x) + \frac{\lambda_{-i}}{\Lambda}V_i(p_i, \zeta, x) \\ \zeta &= \arg\max_{\xi\in G} V_{-i}(p_i, \xi, x). \end{aligned}$$

Given the grid G, we can consider two cases: $p_i < p_{-i}$ (front-side) and $p_i > p_{-i}$ (residual). First suppose that $p_i < p_{-i}$, so firm i is undercutting firm $-i$. When undercutting, instantaneous profits are linearly increasing in p_i. The continuation payoff is locally constant in p_i if $p_i < p_{-i}$ as the search decision is unaffected (following the mean-field approach). Therefore, for all $p_i < p_{-i}$, the profit-maximizing undercut is $p_{-i} - \delta$. Cutting by more than δ lowers the instantaneous front-side profits and, due to potentially increasing the search intensity over time (or reducing the rate at which search intensity decreases) reduces the continuation payoff.

$$\underline{V}(p_{-i}, x) = \frac{1}{\beta + \Lambda}(p_{-i} - \delta)(\alpha_i + (1 - \alpha_i)\bar{\varphi}(p_{-i}, x)) + \frac{\Lambda}{\beta + \Lambda}\int_0^\infty e^{-(\beta+\Lambda)t} W_i(x^t)dt.$$

For the case of $p_i > p_{-i}$, let

$$\bar{V}(\xi, p_{-i}, x) = \frac{1}{\beta + \Lambda}\alpha_i \xi(1 - \bar{\varphi}(\xi, x)) + \frac{\Lambda}{\beta + \Lambda}\int_0^\infty e^{-(\beta+\Lambda)t} W_i(x^t)dt.$$

Define

$$\begin{aligned}\tilde{P}(x, p_{-i}) &= \arg\max_{\xi > p_{-i} \in G} \bar{V}(\xi, p_{-i}, x) \\ \tilde{V}(x, p_{-i}) &= \max_{\xi > p_{-i} \in G} \bar{V}(\xi, p_{-i}, x).\end{aligned}$$

Thus, we can define the forward looking judo price as

$$p_i^*(x) = \max\{g_\omega \in G \setminus \{g_0\} : \underline{V}(g_\omega, x) \geq \tilde{V}(x, p_{-i})\}.$$

Therefore, as long is the gap between the residual maximizer and judo price is non-degenerate, we can write a dynamic version of the best response as

$$BR_i(g_\omega, x) \in \begin{cases} \{g_{\omega-1}\} & \text{if } g_\omega > p_i^*(x) \\ \{g_{\omega-1} \cup \tilde{P}(x, g_\omega)\} & \text{if } g_\omega = p_i^*(x) \\ \tilde{P}(x, g_\omega) & \text{if } g_\omega < p^*(x) \end{cases}$$

Given the inclusion of the continuation payoffs for $\beta > 0$, both the judo prices and residual maximizers are higher than under the myopic case presented in the main paper. However, as long as δ is sufficiently small, the logical structure persists and Edgeworth cycles following from the best response correspondence can persist. If we assume that the mean-field dynamic is also a function of $p_i - p_j$, then there will exist an optimal undercut of $g_{\omega-k}$, for $k > 1$; i.e., rather than undercutting by the minimum, there are larger undercuts (possibly heterogeneous across firms). However, for small enough δ a cycling dynamic still follows. This generally occurs when $|\alpha_i - \alpha_{-i}|$ is sufficiently large, as there is an incentive for the firm with the smaller market share to induce greater search. However, as that firm will also spend a portion of its time as the firm with the higher price, this incentive is tempered, particularly if the time between revisions λ_F is large enough relative to λ_C.

C.2.2 Nonconvergence Results

To show that non-convergence can still persist, we need to show that there exists conditions such that in a MPE, firms select a price vector p that satisfies $c < c^*(p)$. More specifically, we must show that there is a time interval $[T, T+\varepsilon)$ for some positive T and ε such that at $t = T$, $x^T \in \mathcal{N}(e_L)$ and prices are $p^T = (g_m, g_{m'})$ with $m \neq m'$ and, at this price vector, $c < c^*(p)$. This is sufficient for there to exist a $T' \in [T, T+\varepsilon)$ such that, with positive probability, x^T strictly first order stochastically dominates $X^{T'}$ (by Proposition 4) so $X^{T'} \notin \mathcal{N}(e_L)$ and there is non-convergence. Let this neighborhood $\mathcal{N}(e_L)$ be defined as in Lemma 6; i.e., such that $\tilde{P}(x) = \tilde{P}(e_L)$ for $x \in \mathcal{N}(e_L)$.

Suppose that $x^t \in \mathcal{N}(e_L)$ and suppose that prices have converged to some $p = (g_m, g_{m'})$, where without loss of generality, $g_m \leq g'_m$. For the moment, let $g_{m'} = g_m$. In this candidate MPE, firm i's payoff is

$$\int_T^\infty e^{-\beta(t-T)} \alpha_i g_m dt = \frac{1}{\beta} \alpha_i g_m.$$

Consider the one-shot deviation in which firm i lowers its price to g_{m-k} for some $k > 0$ before returning to g_m at its next revision opportunity. Suppose k is such that $c < c^*((g_{m-k}, g_m))$, then by Lemma 1, $\bar{\varphi}(g_m, x^t) \geq \bar{\varphi}(g_m, e_L)$, so the deviation payoff is

$$\frac{1}{\beta + \Lambda} g_{m-k} \left(\alpha_i + (1-\alpha_i)\bar{\varphi}(g_m, e_L)\right) + \frac{\Lambda}{\beta + \Lambda} \int_0^\infty e^{-(\beta+\Lambda)t} W_i(x^{\mathrm{dev(t)}}) dt.$$

Thus, the deviation is profitable if

$$g_{m-k} \left(\alpha_i + (1-\alpha_i)\bar{\varphi}(g_m, e_L)\right) + \Lambda \int_0^\infty e^{-(\beta+\Lambda)t} W_i(x^{\mathrm{dev(t)}}) dt$$
$$> \alpha_i g_m + \Lambda \int_0^\infty e^{-(\beta+\Lambda)t} W_i(x^{\mathrm{t}}) dt.$$

Because undercutting weakly increases search,

$$\int_0^\infty e^{-(\beta+\Lambda)t} W_i(x^{\mathrm{dev(t)}}) dt \leq \int_0^\infty e^{-(\beta+\Lambda)t} W_i(x^{\mathrm{t}}) dt,$$

so this deviation is profitable if

$$g_{m-k} \left(\alpha_i + (1-\alpha_i)\bar{\varphi}(g_m, e_L)\right) > \alpha_i g_m,$$

which simplifies to

$$\bar{\varphi}(g_m, e_L) = \varphi(g_m - e_L) > \frac{\alpha_i}{1-\alpha_i} \left(\frac{g_m - g_{m-k}}{g_{m-k}}\right).$$

Hence, such a deviation is profitable and there is not convergence (as in Theorem 1) if either (i) the grid is sufficiently fine, (ii) α_i is sufficiently small, or (iii) the probability of search at observed price g_m and threshold e_L is sufficiently large. Thus, imposing some additional restrictions on the shape of $\varphi(\cdot)$ is sufficient to generate a non-convergence result in the forward looking case.

C.2.3 Convergence Results

It is relatively straightforward to argue that the equilibrium dynamics in Theorem 2 and Proposition 5 can be constructed as the outcomes of a Markov perfect equilibrium with forward looking firms (though these dynamics need not be unique). Under C1$'$, the firms can always guarantee $x^t \to e_L$ as $t \to \infty$. Set $x = e_L$. If $\varphi(1-\tau_L)$ is sufficiently small, then a MPE resembling the Diamond paradox is the unique outcome for a sufficiently small discount rate β (see the argument for Proposition 5 below). If, on the other hand, $\varphi(1-\tau_L)$ is sufficiently large, then a focal point equilibrium of the from $(1,1)$ cannot exist, as a one-shot deviation (as determined by the stochastic Poisson process) to $p_i = 1-\delta$ yields

$$\frac{1}{\beta+\Lambda} g_{M-1}\left(\alpha_i + (1-\alpha_i)\varphi(1-e_L)\right) + \frac{\Lambda}{\beta+\Lambda}\alpha_i.$$

Recalling that $g_M = 1$ (the monopoly price), the above deviation is profitable if

$$\begin{aligned} g_{M-1}\left(\alpha_i + (1-\alpha_i)\varphi(1-e_L)\right) > \alpha_i & \\ \varphi(1-e_L) > \frac{\alpha_i}{1-\alpha_i}\left(\frac{\delta}{g_{M-1}}\right). \end{aligned}$$

Hence, there is no kinked demand equilibrium for $\varphi(1, e_L)$ large (a sufficiently small δ guarantees that the right-hand side of the above is less than 1). By an analogous argument, the same holds for kinked demand equilibria of the form $p = (1, g_{M-k})$.

Under the conditions set in Proposition 5, $\varphi(1-\tau_L) \to 0$. Hence, for discount factor β, at $p = (1,1)$ and $x \in \mathcal{N}(e_L)$, firm i's value function at this pricing profile is $\frac{1}{\beta}\alpha_i$. A deviation to $g_m \in G \setminus \{1\}$ yields at most

$$\begin{aligned} &\max_{g_m \in G\setminus\{1\}} \frac{1}{\beta+\Lambda} g_m\left(\alpha_i + (1-\alpha_i)\bar{\varphi}(1, e_L)\right) + \left(\frac{\Lambda}{\beta+\Lambda}\right)\frac{1}{\beta}\alpha_i \\ &= \max_{g_m \in G\setminus\{1\}} \frac{1}{\beta+\Lambda}\alpha_i g_m + \left(\frac{\Lambda}{\beta+\Lambda}\right)\frac{1}{\beta}\alpha_i \\ &< \frac{1}{\beta+\Lambda}\alpha_i + \left(\frac{\Lambda}{\beta+\Lambda}\right)\frac{1}{\beta}\alpha_i \\ &= \frac{1}{\beta}\alpha_i. \end{aligned}$$

Thus, Proposition 5 is satisfied for all $\beta > 0$.

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