Dynamic Price Competition and Evolutionary Behavior with Search: Online Appendix*

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Online Appendix A

A.1 Extension to N Firms

The results Sections 2 and 3 can be readily generalized to the N firm case. The proofs of the Propositions and Theorems are built upon the judo prices and residual maximizers. The residual maximizers are independent of the number of firms. Hence, the results an be immediately generalized by augmenting the definition of the judo price to the N firm case.

Let $p^t = (p_1^t, \ldots, p_n^t)$, where p_{-i}^t has the typical interpretation. Let α_i denote the mass of consumers that observe firm i's price with $\sum_{j=1}^N \alpha_j = 1$. We will now construct the demand facing each firm as a function of the price vector p^t and the distribution of consumers' thresholds x^t. Note that if a firm i's price is lower than all of the other firms, then it will serve the consumers that initially observe p_i^t as well as all of the consumers that observe other prices and search. If firm i and $b-1$ other firms have the lowest price then firm i will serve the consumers that initially observe p_i^t and do not search as well as $1/b$ of all of the consumers that search. Finally, if firm i does not have the lowest price, then it will serve only the consumers that initially observe p_i^t and do not search. Let $I(p^t) = \{i : p_i^t = \min_j p_j^t\}$ and $J(p^t) = \{i : p_i^t > \min_j p_j^t\}$. Then we may express the demand facing each firm i as

$$D_i\left(p^t, x^t\right) = D\left(p_i^t\right) \times \begin{cases} \alpha_i + \sum_{j\neq i} \alpha_j \bar{\varphi}(p_j^t, x^t) & \text{if } p_i^t < \min_{j\neq i} p_j^t \\ \alpha_i + \frac{1}{|I(p^t)|} \sum_{j\in J(p^t)} \alpha_j \bar{\varphi}(p_j^t, x^t) & \text{if } p_i^t = \min_{j\neq i} p_j^t \\ \alpha_i(1 - \bar{\varphi}(p_i^t, x^t)) & \text{if } p_i^t > \min_{j\neq i} p_j^t. \end{cases}$$

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The front-side and residual profits are now

$$\pi_i^F(p,x) = p_i D(p_i)\left(\alpha_i + \sum_{j\neq i}\alpha_j\bar{\varphi}(p_j,x)\right)$$
$$\pi_i^R(\xi,x) = \xi D(\xi)\alpha_i(1-\bar{\varphi}(\xi,x))$$

and the residual maximizer $\tilde{P}(x)$ (and $\tilde{P}(x,G)$) retain their original definitions and structures.

The best response correspondence with N firms is nearly identical to that when there are only two firms. The main difference is that the notion of the judo price is very different when there are multiple firms. Rather than a firm's preference between monopolizing its residual demand and undercutting being determined entirely by the price that it must set in order to undercut, this preference is determined by all the prices set by the competing firms, not just the lowest price. The reason is that as some firms charge higher prices, more of the consumers that observe their prices will search, thereby creating a higher incentive for the remaining firms to set the lowest price in order to attract those searching consumers.

We generalize the notion of the judo price to the judo correspondence, defined analogously to the judo price with two firms. For each pair of firms i, j, let p_{-ij} denote the vector of prices for the firms that are neither i nor j. For any vector v, let $\underline{\underline{p}}(v) = \min v$ denote the smallest component of v. Define the conditional judo price for firm i relative to firm j given the vector p_{-ij} as

$$p_{ij}^*(p_{-ij},x) = \sup\Big\{\xi \leq \xi^m : \min\Big\{\xi, \underline{\underline{p}}(p_{-ij})\Big\} D\left(\min\Big\{\xi, \underline{\underline{p}}(p_{-ij})\Big\}\right)\Big(\alpha_i + \alpha_j\bar{\varphi}(\xi,x) + \sum\nolimits_{h\neq i,j}\alpha_h\bar{\varphi}(p_h,x)\Big) < \max_{p_i}\alpha_i p_i D(p_i)(1-\bar{\varphi}(p_i,x))\Big\}.$$

If $p_{ij}^*(p_{-ij},x) = \xi^m$ and

$$\underline{\underline{p}}(p_{-ij})D\left(\underline{\underline{p}}(p_{-ij})\right)\left(\alpha_i + \alpha_j\bar{\varphi}(\xi^m,x) + \sum\nolimits_{h\neq i,j}\alpha_h\bar{\varphi}(p_h,x)\right) < \max_{p_i}\alpha_i p_i D(p_i)(1-\bar{\varphi}(p_i,x)),$$

then set $p_{ij}^*(p_{-ij},x) = \infty$. Given these conditional judo prices, define firm i's judo correspondence to be

$$P_i^*(x) = \bigcup\nolimits_{j\neq i}\bigcup\nolimits_{p_{-ij}\in[0,\xi^m]}(p_{ij}^*(p_{-ij},x), p_{-ij}).$$

The constrained Judo prices (on G) are defined analogously. Note that one immediate difference between the judo price for a duopoly and the conditional judo price is that the conditional judo price may be strictly less than the monopoly price while some of the other prices are below the search cost c, meaning that the firm would maximize its profits by choosing a price below the search cost. The reason is that firms pricing above the search cost induce some amount of search from consumers, and so a firm may wish to undercut other firms that have prices below c in order to attract the searching consumers. This was not possible with only two firms, as if the single other firm has a price below the cost of search, then there are no searching consumers to attract and thus there is no incentive for a firm to undercut that price.

Given this Judo correspondence, under the conditions outlined in Proposition 2, the best response correspondence for N firms is qualitatively similar to the two firm case (each firm will either undercut or monopolize its residual demand). Formally:

$$R_i(p_{-i}, x) = \begin{cases} \{g_{\omega-1}\} & \text{if} \quad p_{-i} \in \mathrm{epi}_S P_i^*(x,G) \\ \{g_{\omega-1}\} \cup \tilde{P}(x,G) & \text{if} \quad p_{-i} \in P_i^*(x,G) \\ \tilde{P}(x,G) & \text{if} \quad p_{-i} \in \mathrm{hyp}_S P_i^*(x,G), \end{cases}$$

where $\mathrm{epi}_S P_i^*(x,G)$ and $\mathrm{hyp}_S P_i^*(x,G)$ denote the respective strict epigraph and strict hypograph (subgraph) of $P_i^*(x,G)$. The critical judo price is now $p^*(x) = \max_i \{\inf \mathrm{epi}_S P_i^*(x)\}$ (as defined independent of G with the analogous definition of $p^*(x,G)$ on G) and the subsequent proofs follow analogous arguments to the two firm case.

A.2 Assumption on the Observed Price α_i

The main text assumes that α_i is constant, so the firm that a consumer observes is independent of their previous purchase. Though this assumption has no bearing on the results (in the myopic firm specification), it can alter the interpretation of Proposition 4. Before discussing the alternative implication, we first briefly illustrate with an example that this assumption is reasonable in some markets. Consider the retail gasoline market and suppose that there is a road with two gas stations. At one end of the road is the consumer's home and at the other end is the consumer's place of work. The gas stations are at intermediate locations. If a consumer is traveling from home to work, that consumer observes a different gas station's price than if the consumer is traveling from work to home.

Suppose now that α_i is determined by the distribution of purchases, so at each time, α_i is determined by i's previous sales. This would simply move the judo price at each instant, but none of the proofs of the propositions rely on a constant α_i. What matters is the value of α_i at the time that firm i is called to move according to its Poisson process. The best response correspondence of Proposition 2 still governs the pricing decision. Under Proposition 4, the firm with the largest 'installed base' has the stronger incentive to relent (the higher judo price). With a constant α_i and in the context of retail gasoline markets, we can think of the firms with the larger α_i as the 'major brands' and the others as the 'independent brands.' The empirical literature is consistent with the results of Proposition 4: major brands are the first to relent in retail gasoline markets characterized by Edgeworth cycles (Noel, 2007; Atkinson, 2009; Isakower and Wang, 2014).

In the forward-looking case (See Appendix C.2 below), this assumption has a more significant impact on the results. If α_i is determined by previous sales, then there is a greater incentive to boost current sales as this will induce an increased subsequent residual demand. Hence, there is a greater incentive to undercut. However, the judo price is still well defined, so there is still an eventual incentive to relent when price is above marginal cost and the cycles can persist as described.

Online Appendix B

B.1 First Order Stochastic Dominance and Residual Maximizers

Suppose that (i) $D(\cdot)$ is defined such that if all consumers have a single threshold τ_k, then the residual maximizer is increasing in that τ_k, and (ii) σ has a very large probability of taking on values in the range $(\tilde{P}(x) - \tau_{k+1}, \tilde{P}(x) - \tau_{k-1})$ and a very small probability of taking on other values. Consider two distributions x and x', where under both x and x', half of the consumers have τ_L. Under x, the remaining half of the consumers have threshold τ_k. Under x', the remaining half of the consumers threshold τ_{k-1}.

Thus, under x, there are two candidate residual maximizers: a low maximizer that attempts to serve both types of consumers and a high maximizer, which induces a very high probability that the τ_k consumers search and maximizes the revenue from the τ_L consumers. Now consider x'. At the price $\tilde{P}(x')$, the τ_{k-1} consumers search with near certainty. As such, the low maximizer must be smaller under x', while the high maximizer is the same regardless of the distribution. As $\xi D(\xi)$ is increasing in ξ, this lower maximizer may yield a smaller profit under x' than $\tilde{P}(x)$ yields under x. If the difference is large enough, then the residual maximizer under x' would be the high maximizer. Hence, with consumers searching more, the residual maximizer is higher.

B.2 Uniformly Distributed Noise

To show that the upper bound of the cycle is higher under x than x' when x first order stochastically dominates x', it must be that the smallest residual maximizer is under x is greater than the largest residual maximizer under x'. Hence, it is sufficient to show that $\frac{\partial \pi_i^R(\xi,x)}{\partial \xi} > \frac{\partial \pi_i^R(\xi,x')}{\partial \xi}$ for all ξ. Note that

$$\frac{\partial \pi_i^R(\xi,x)}{\partial \xi} = D(\xi)(1-\bar{\varphi}(\xi,x)) + \xi\frac{\partial D(\xi)}{d\xi}(1-\bar{\varphi}(\xi,x)) - \xi D(\xi)\frac{\partial \bar{\varphi}(\xi,x)}{\partial \xi},$$

where $\frac{\partial \bar{\varphi}(\xi,x)}{\partial \xi} = \frac{1}{\text{range}\,\text{supp}\,\varphi} = z$ for some constant $z > 0$. Hence, the upper bound of the cycle is larger under x than x' if

$$\begin{aligned} D(\xi)(1-\bar{\varphi}(\xi,x)) &+ \xi\frac{\partial D(\xi)}{d\xi}(1-\bar{\varphi}(\xi,x)) - \xi D(\xi)z \\ &\geq D(\xi)(1-\bar{\varphi}(\xi,x')) + \xi\frac{\partial D(\xi)}{d\xi}(1-\bar{\varphi}(\xi,x')) - \xi D(\xi)z, \end{aligned}$$

which simplifies to

$$\left(D(\xi) + \xi\frac{\partial D(\xi)}{d\xi}\right)(\bar{\varphi}(\xi,x) - \bar{\varphi}(\xi,x')) \leq 0.$$

The first term is nonnegative for all $\xi \leq \xi^m$ and strictly positive for all $\xi < \xi^m$. The second term is strictly negative by Lemma 1 as x first order stochastically dominates x'. Thus, when the noise is uniformly distributed, $\inf \tilde{P}(x) \geq \sup \tilde{P}(x')$ and strictly so when interior.

B.3 The Diamond Paradox

Consider the following two stage game set in the market described above. In the first stage, the firms simultaneously set their prices. In the second stage, each consumer visits one of the firms at random, with firm 1 being visited with probability α and firm 2 with probability $1-\alpha$. After visiting a firm, the consumer learns the price set by that firm. The consumers may then incur a cost $c > 0$ in order to learn the price of the other firm. Finally, the consumers purchase $q = D(\xi)$ from the firm with the lowest observed price ξ.

In order to study the equilibrium of this model, it is necessary to specify the beliefs that consumers have regarding the unobserved price. We follow the literature in assuming that the consumers form rational expectations regarding the price (or distribution of prices) that the other firm has set, correctly anticipating the equilibrium price (or distribution of prices).

Definition 1. *A pure (mixed) strategy Nash equilibrium of this game is a price p_i (distribution of prices F_i) for each firm and a search rule for the consumers such that*

1. *p_i (F_i) maximizes firm i's profits conditional the other firms price p_{-i} (distribution of prices F_{-i}) and the search rule of the consumers.*
2. *The search rule used by the consumers is optimally derived from the firms prices p (distributions of prices F).*

Provided that consumers correctly anticipate the firms' strategies in equilibrium, this game possesses a unique Nash equilibrium in which both firms charge the monopoly price and no consumers search. This outcome is referred to as the *Diamond paradox*. Intuitively, if consumers do not search, then firms will monopolize their consumer base. Given that firms are charging the monopoly price, consumers cannot benefit from search. We will now show formally that this is the unique equilibrium.

Proposition B1. *There is a unique Nash equilibrium in which consumers correctly anticipate the firms' strategies. In this equilibrium, each firm i sets $p_i = \xi^m$, and consumers search if and only if they observe a price $p > \xi^m + c$.*

We prove this result with the help of three lemmas. The first verifies that the proposed strategies constitute an equilibrium. The second shows that there are no other pure strategy Nash equilibria. The final shows that in any (mixed strategy) Nash equilibrium, the supremum of the support of each firm's strategy is identical.

Lemma B1. *The strategies $p_1 = p_2 = \xi^m$ for the firms and search if and only if the observed price $p > \xi^m + c$ for consumers constitutes a Nash equilibrium.*

Proof. Suppose that all consumers expect that the unobserved price is set at the monopoly price ξ^m. If a consumer observes a price $p \leq \xi^m + c$, then she will never find it optimal to search as she anticipates that the unobserved price is weakly higher than the observed price less the search cost. She is therefore unwilling to incur the cost of search. Thus, given that both firms are charging the monopoly price, the consumer decision to not search is optimal. As the consumers will not search (except possibly if the price they observe is greater than $\xi^m + c$), the firms will find it optimal to charge the monopoly price. Clearly, charging a higher price cannot possibly increase profits and setting a lower price cannot either since the consumers that visit the other firm first will not search, so neither firm can attract the other's customers. Therefore, this strategy profile is an equilibrium. □

Lemma B2. *There is no pure strategy Nash equilibrium in which a firm i sets a price $p_i < \xi^m$.*

Proof. Suppose to the contrary that there exists a pure strategy equilibrium in which $p_1 < \xi^m$. Given that consumers must correctly anticipate the firms' strategies, the optimal search rule when observing a price p_i is to search if and only if $p_i > p_{-i} + c$. If firm i's price is such that $p_i > p_{-i} + c$, then i's consumers will search and purchase from firm $-i$. Hence, firm i earns zero profits. By setting a price $p_i = p_{-i} + c$, firm i earns positive profits. Therefore, each firm's price is such that $p_i \leq p_{-i} + c$. It follows that each firm's profit maximizing price is $\rho_i = \min\{\xi^m, p_{-i} + c\}$. In particular, this pricing strategy implies that $p_2 > p_1$, which further implies that $\rho_1 > p_1$, violating p_1 as an equilibrium strategy. □

Before stating and proving Lemma B3, we introduce some notation. Denote by F_i the cumulative distribution function that defines an equilibrium price distribution for firm i, and denote by $\underline{\gamma}_i$ and $\bar{\gamma}_i$ the infimum and supremum of the support of F_i. Let $\hat{p}_i$ denote the average price set by firm i when employing the mixed strategy F_i. Note that a consumer that visits firm i will find it beneficial to search if and only if

$$\max\{0, v(D(\min\{p_i, \hat{p}_{-i}\})) - \min\{p_i, \hat{p}_{-i}\}D(\min\{p_i, \hat{p}_{-i}\})\} - c > \max\{0, v(D(p_i) - p_i D(p_i)\}$$

Hence, a consumer will search if and only if both $v(D(\hat{p}_{-i})) \geq \hat{p}_{-i}D(\hat{p}_{-i})) + c$ and $p_i > \hat{p}_{-i} + c$.

Lemma B3. $\bar{\gamma}_1 = \bar{\gamma}_2 \leq \xi^m$.

Proof. We first show that $\bar{\gamma}_1 \leq \xi^m$. By symmetry, this will imply that $\bar{\gamma}_2 \leq \xi^m$. When choosing a price p_1, firm 1's profit is

$$\pi_1(p_1, F_2) = \begin{cases} p_i D(p_i)(\alpha + (1-\alpha)(1 - F_2(\max\{p_1, \widehat{p}_1 + c\}))) & \text{if } p_1 \leq \hat{p}_2 + c \\ p_i D(p_i)(\alpha(1 - F_2(p_1)) + (1-\alpha)(1 - F_2(\max\{p_1, \hat{p}_1 + c\}))) & \text{if } p_1 > \hat{p}_2 + c \end{cases}$$

As the term multiplying the revenue function is weakly decreasing in p_1 and $p_i D(p_i)$ is quasiconcave with unique maximizer ξ^m (by B1), it must be that $\pi_1(\xi^m, F_2) > \pi_1(p_1, F_2)$ for any $p_1 > \xi^m$. Therefore, $\bar{\gamma}_1 \leq \xi^m$.

Next, we show that $\bar{\gamma}_1 = \bar{\gamma}_2$. Suppose to the contrary that $\bar{\gamma}_1 > \bar{\gamma}_2$. We proceed in two cases.

<u>Case 1</u>: $\bar{\gamma}_1 > \hat{p}_2 + c$. When firm 1 sets a price near $\bar{\gamma}_1$, all of its consumers will search with certainty. Moreover, because $\bar{\gamma}_1 > \bar{\gamma}_2$, the consumers will find that $p_2 < p_1$, so firm 1 will earn a profit of zero. By setting a price $p_1' = \hat{p}_2 + c$, firm 1 would be guaranteed a positive profit, violating F_1 as an equilibrium strategy.

<u>Case 2</u>: $\bar{\gamma}_1 \leq \hat{p}_2 + c$. For any $p_1 \in [\bar{\gamma}_2, \bar{\gamma}_1]$, none of firm 1's consumers search, and any consumers that search from firm 2 will not purchase from firm 1. Thus, $p_1' = \bar{\gamma}_1$ earns a strictly higher profit than any $p_1 \in [\bar{\gamma}_2, \bar{\gamma}_1)$. Hence, F_1 must place zero mass on the interval $[\bar{\gamma}_2, \bar{\gamma}_1)$.

If $\bar{\gamma}_2 > \hat{p}_1 + c$, then any price p_2 near $\bar{\gamma}_2$ is strictly less profitable for firm 2 than any $p_2' \in (\bar{\gamma}_2, \bar{\gamma}_1)$, since p_2' is closer to the monopoly price and does not increase the probability that a customer that visits firm 2 and searches will find a lower price at firm 1. Thus, it must be that $\bar{\gamma}_2 \leq \hat{p}_1 + c$. As the highest price charged by each firm is no larger than the expected price of its competitor plus the cost of search, no consumers ever find it beneficial to search. Therefore, $p_i = \bar{\gamma}_i$ is strictly more profitable than any $p_i' < \bar{\gamma}_i$, since $\bar{\gamma}_i$ is closer to the monopoly price. Thus, each firm must be using a pure strategy with $p_i = \bar{\gamma}_i$.

By Lemmas B1 and B2, $\underline{\gamma}_1 = \underline{\gamma}_2 = \bar{\gamma}_1 = \bar{\gamma}_2 = \xi^m$, which contradicts the assumption that $\bar{\gamma}_1 > \bar{\gamma}_2$. $\square$

Proof of Proposition B1.

Proof. By Lemma B3, the firms must be employing pure strategies in equilibrium. By Lemmas B1 and B2, the only pure strategy Nash equilibrium is monopoly pricing, in which case there is zero search. $\square$

Online Appendix C

C.1 Discussion of Myopic Firms

A potential concern is that the results are driven by the choice to model firms as myopic payoff maximizers rather than forward looking. However, this assumption is motivated by the primitive assumption that firms do not have knowledge of the process by which consumer behavior changes over time. Indeed, there is no assumption (or need to assume) that consumers understand this process, and so even if the managers of the firms are themselves consumers, this need not impart any such knowledge. Despite not understanding the process by which consumers update their strategies, it is possible for firms to observe (or at least estimate) the current state, e.g., via market research, consumer surveys, or focus groups. Given sufficient data, the firms can accurately predict the current state, though no amount of data is sufficient to identify the consumer process. Without an understanding of the consumer dynamic, it is not possible for firms to form meaningful expectations as to how the distribution of consumers' price thresholds will change in response to the prices that are set. For example, firms may recognize the possibility that the consumers act as they do in the model, while at the same time anticipating that consumers search more (lower their thresholds) whenever prices are high. The latter belief could be derived from the anticipation that consumers strategically search more in response to high prices so as to induce firms to lower their prices in the future.

In addition to uncertainty regarding the nature of the consumer dynamic, there is an additional lack of information regarding the rate at which the consumers change their strategies. With so little information, it is unreasonable to expect that firms anticipate the consumers' reactions to their prices. Any prior that the firms have over the possible consumer dynamics ought to be uninformative, which would assign an equal probability to any response and its opposite, leading to a belief that the consumer dynamic will remain unchanged in expectation. While the dynamic may be fixed in expectation, the realization of the distribution of consumers' price thresholds will almost certainly change before either firm gets another opportunity to change its price. As such, forward looking optimization would require maximizing the net present expected value of the profits, which requires further anticipation of each firm's response in all possible states and all possible posterior beliefs about the consumer dynamic in all future periods. Given the staggering variance of predictions about future behavior and the immense computational burden, it seems reasonable to assume that firms either are unable to determine the optimal pricing strategy to maximize their stream of profits and instead opt to take the more simple approach of maximizing short run profits.

C.2 Intuition for Extension to Forward Looking Firms

The primary benefit of the myopic firm assumption is the precision it grants in identifying the short-run dynamics. Here, we informally illustrate that the general patterns identified in the main results can (with some minor additional assumptions) carry over to the forward-looking case, though much of the precision is lost. Hence, the main results are primarily driven by the demand side of the market rather than the myopic firm assumption. A complete formal characterization of the forward looking case is left for future work.

Let λ_i denote firm i's Poisson parameter and $\beta > 0$ the common discount rate. Throughout, profits refer to expected profits and we will assume that $\|G\|$ is sufficiently small with respect to the search cost c so that for any $p = (g_m, g_{m-1})$ and $m \geq 1$, $c > c^*(p)$. That is, search has a negative expected utility when the firms' prices are adjacent.

C.2.1 Nonconvergence Results

To show that non-convergence can still persist, we need to show that there exists conditions such that in a MPE, firms select a price vector p that satisfies $c < c^*(p)$. More specifically, we must show that there is a time interval $[T, T+\varepsilon)$ for some positive T and ε such that at $t = T$, $x^T \in \mathcal{N}(e_L)$ and prices are $p^T = (g_m, g_{m'})$ with $m \neq m'$ and, at this price vector, $c < c^*(p)$. This is sufficient for there to exist a $T' \in [T, T+\varepsilon)$ such that, with positive probability, x^T strictly first order stochastically dominates $X^{T'}$ (by Proposition 5) so $X^{T'} \notin \mathcal{N}(e_L)$ and there is non-convergence. Let this neighborhood $\mathcal{N}(e_L)$ be defined as in Lemma 6; i.e., such that $\tilde{P}(x) = \tilde{P}(e_L)$ for $x \in \mathcal{N}(e_L)$.

Suppose that $x^t \in \mathcal{N}(e_L)$ and suppose that prices have converged to some $p = (g_m, g_{m'})$, where without loss of generality, $g_m \leq g'_m$. For the moment, let $g_{m'} = g_m$. In this candidate MPE, firm i's payoff is

$$\int_T^\infty e^{-\beta(t-T)} \alpha_i g_m D(g_m) dt = \frac{1}{\beta} \alpha_i g_m D(g_m).$$

Consider the one-shot deviation, lasting an expected time interval of λ_i^{-1}, in which firm i lowers its price to g_{m-k} for some $k > 0$ before returning to g_m at its next revision opportunity. This deviation yields a payoff of

$$\int_T^{T+\lambda_i^{-1}} e^{-\beta(t-T)} g_{m-k} D(g_{m-k}) \left(\alpha_i + (1-\alpha_i)\bar{\varphi}(g_m, x^t)\right) dt + \frac{e^{-\beta\lambda_i^{-1}}}{\beta} \alpha_i g_m D(g_m).$$

Suppose k is such that $c < c^*((g_{m-k}, g_m))$, then by Lemma 1, $\bar{\varphi}(g_m, x^t) \geq \bar{\varphi}(g_m, e_L)$, so the first term of the deviation payoff is bounded below by

$$\frac{1 - e^{-\beta\lambda_i^{-1}}}{\beta} g_{m-k} D(g_{m-k}) \left(\alpha_i + (1-\alpha_i)\bar{\varphi}(g_m, e_L)\right).$$

Therefore, this deviation is profitable if

$$g_{m-k} D(g_{m-k}) \left(\alpha_i + (1-\alpha_i)\bar{\varphi}(g_m, e_L)\right) > \alpha_i g_m D(g_m),$$

which simplifies to

$$\bar{\varphi}(g_m, e_L) = \varphi(g_m - e_L) > \frac{\alpha_i}{1-\alpha_i}\left(\frac{g_m D(g_m) - g_{m-k}D(g_{m-k})}{g_{m-k}D(g_{m-k})}\right).$$

Hence, such a deviation is profitable and there is not convergence (as in Theorem 1) if either (i) the grid is sufficiently fine, (ii) α_i is sufficiently small, or (iii) the probability of search at observed price g_m and threshold e_L is sufficiently large. Thus, imposing some additional restrictions on the shape of $\varphi(\cdot)$ is sufficient to generate a non-convergence result in the forward looking case.

C.2.2 Convergence Results

It is relatively straightforward to argue that the equilibrium dynamics in Theorem 2 and Proposition 6 can be constructed as the outcomes of a Markov perfect equilibrium with forward looking firms (though these dynamics need not be unique). Under C1′, the firms can always guarantee $x^t \to e_L$ as $t \to \infty$. Set $x = e_L$. If $\varphi(\xi^m - \tau_L)$ is sufficiently small, then a MPE resembling the Diamond paradox is the unique outcome for a sufficiently small discount rate β (see the argument for Proposition 6 below). If, on the other hand, $\varphi(\xi^m - \tau_L)$ is sufficiently large, then a focal point equilibrium of the from (ξ^m, ξ^m) cannot exist, as a one-shot deviation (as determined by the stochastic Poisson process) to $p_i = g_{M-1}$ yields

$$\frac{1-e^{-\beta\lambda_i^{-1}}}{\beta} g_{M-1}D(g_{M-1})\left(\alpha_i + (1-\alpha_i)\varphi(\xi^m - e_L)\right) + \frac{e^{-\beta\lambda_i^{-1}}}{\beta}\alpha_i \xi^m D(\xi^m).$$

Recalling that $\xi^m = g_M$, the above deviation is profitable if

$$\begin{aligned} g_{M-1}D(g_{M-1})\left(\alpha_i + (1-\alpha_i)\varphi(\xi^m - e_L)\right) &> \alpha_i g_M D(g_M) \\ \varphi(\xi^m - e_L) &> \frac{\alpha_i}{1-\alpha_i}\left(\frac{g_M D(g_M) - g_{M-1}D(g_{M-1})}{g_{M-1}D(g_{M-1})}\right). \end{aligned}$$

Hence, there is no kinked demand equilibrium for $\varphi(\xi^m, e_L)$ large (a sufficiently small $\|G\|$ guarantees that the right-hand side of the above is less than 1). By an analogous argument, the same holds for kinked demand equilibria of the form $p = (g_M, g_{M-k})$. Following an analogous approach to Maskin and Tirole (1988), a best response analogous to Proposition 2 can be constructed that yields a MPE with Edgeworth cycles. The proof would also proceed in a manner similar to Maskin and Tirole (1988) with the exception that the Judo price still dictates the lower bound of the Edgeworth cycle rather than the marginal cost.[1]

Under the conditions set in Proposition 6, $\varphi(\xi^m - \tau_L) \to 0$. Hence, for discount factor β, at $p = (\xi^m, \xi^m)$ and $x \in \mathcal{N}(e_L)$, firm i's value function at this pricing profile is

$$\frac{1}{\beta}\alpha_i \xi^m D(\xi^m).$$

A deviation to $g_m \in G \setminus \xi^m$ yields at most

$$\begin{aligned}
&\max_{g_m \in G \setminus \xi^m} \frac{1 - e^{-\beta\lambda_i^{-1}}}{\beta} g_m D(g_m)\left(\alpha_i + (1-\alpha_i)\bar{\varphi}(\xi^m, e_L)\right) + \frac{e^{-\beta\lambda_i^{-1}}}{\beta}\alpha_i \xi^m D(\xi^m) \\
&= \max_{g_m \in G \setminus \xi^m} \frac{1 - e^{-\beta\lambda_i^{-1}}}{\beta}\alpha_i g_m D(g_m) + \frac{e^{-\beta\lambda_i^{-1}}}{\beta}\alpha_i \xi^m D(\xi^m) \\
&< \frac{1 - e^{-\beta\lambda_i^{-1}}}{\beta}\alpha_i \xi^m D(\xi^m) + \frac{e^{-\beta\lambda_i^{-1}}}{\beta}\alpha_i \xi^m D(\xi^m) \\
&= \frac{1}{\beta}\alpha_i \xi^m D(\xi^m).
\end{aligned}$$

Thus, Proposition 6 is satisfied for all $\beta > 0$.

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[1] See the working paper version, Maskin and Tirole (1985), for the proof.