Composition Effects in Platforms with Population Heterogeneity*

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Abstract

I develop a duopoly model of competition between platforms, incorporating users with heterogeneous preferences over both the platforms' characteristics and the presence of other users. Hence, platforms are concerned with both the number and composition of users. The model yields novel representations of heterogeneity, size effects, and composition effects. I use these representations to decompose the relationship between the price and the size and composition of a platform. Prices need not be monotonic in the size of the installed base and profitability can similarly vary inversely. I identify conditions under which prices are increasing, decreasing, or unchanging in platform size. Given that users care about composition, nonpricing strategies to cultivate platform composition emerge when platforms cannot sufficiently leverage price discrimination. Composition effects and cultivation reframe the dominant-firm fringe-firm paradigm and explain the presence of multi-product firms in platform markets, such as online dating.

May 16, 2025

JEL Classification: D23, Z12.

Keywords: club goods, extremism, identity-based organizations.

*I am grateful to Jean-Paul Carvalho, for invaluable comments and guidance. This paper has benefited greatly from comments by Jiawei Chen, Yong Chao, Linda Cohen, Daniel Grossman, Joshua Hall, Justin Johnson, Christoffer Koch, Paul Lombardi, Michael McBride, Jennifer Muz, Marius Schwartz, Stergios Skaperdas, Angela Vossmeyer, and Timothy Wong. I also thank participants at the 16^{th} annual International Industrial Organization Conference and seminar participants at West Virginia University. All errors are my own.

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1 Introduction

As technology evolves and the world becomes increasingly interconnected, a better understanding of network externalities becomes increasingly important. Meta's market cap is (approx.) $1.65 trillion, Match Group Inc.'s is over $7 billion, and the various online videogaming platforms total over $1 trillion (Yahoo Finance, 2025).[1] A user's willingness to pay for access to a platform depends on the number of other users—this statement is true but lacking. A user's willingness to pay for access to a platform also depends on the characteristics of other users. Similarly, willingness to participate in networks such as open source software development communities depends on user composition.

Network size effects are very well studied.[2] Less attention has been devoted to understanding how the composition of the installed base influences the users' valuations of platforms and thus platforms' responses to user composition effects.[3] The effects of not accounting for composition are consequential. First, ignoring composition effects distorts the pricing strategies of platforms, creating a wedge between expected and observed prices. Second, ignoring composition effects masks a second decision making margin for platforms: product placement (or in competitive environments, product differentiation) over the network externality. If composition is valuable, then platforms will actively engage in efforts to both influence their composition and shape user preferences. When unable to engage in price discrimination to influence user choice, the platforms engage in nonpricing strategies to exert control over their composition. These nonpricing strategies interact with pricing strategies, altering our understanding of platform behavior. This paper develops a tractable representation of heterogeneity, which precisely characterizes the changes in predicted behavior when platforms internalize composition effects. Given that there are differences between traditional industries and these digital platforms in the new economy, new policy protocols must be developed to address the potential market failures associated with platforms and the industries that interact with them.

[1] These values are current as of May 13 and are sourced from Yahoo Finance. Online videogaming platforms include EA, Microsoft, Nintendo, Roblox, Sony, and Tencent.

[2] See Shy (2001), Farrell and Klemperer (2007), Birke (2009), and Shy (2011) for surveys.

[3] Exceptions analyzing composition effects include games played on topological networks. See, for example, Goyal (2009) and Jackson (2010). Unlike the traditional topological models, I assume neither a binary nor symmetric graph (many other works relax those assumptions as well). Within the marketing literature, Algesheimer *et al.* (2005) empirically demonstrates the effects of composition using European automotive clubs. Basu (1989) is also noteworthy in his discussion of status goods, such as awards, modeling the consumption profile of commodities for which, "[. . .] the utility [. . .] depends on who its other recipients (or consumers) are" Basu (1989, p. 654).

This paper develops a model of duopoly competition between single-sided platforms (platforms that charge a single price to all users). Consumers are endowed with heterogeneous characteristics and preferences, including heterogeneous preferences over network composition. I use the model to analyze how incorporating composition alters our understanding of platforms and competition in these industries.

The model provides two methodological contributions. First, I express a gross network valuation function with two arguments: an unadjusted network size and an adjusted network composition. This representation allows for a clean comparative static analysis that decomposes size and composition effects suitable for empirical estimation. With this decomposition, I can precisely characterize the effects of changes in the installed base on prices and the effects of changes in nonpricing strategies on composition and therefore prices. Second, I develop a tractable metric for measuring heterogeneity on a platform. This *heterogeneity-weighted network effect* reduces the high dimensional problem of measuring heterogeneity into a single value. For example, in online dating users vary by characteristics such as age, race, religion, gender, sexual orientation, and location. The value of adding an additional user depends on both the user's characteristics and the characteristics of current users. The heterogeneity-weighted network effect aggregates this information to a single real value. These two features are built using only observable information, allowing for both theoretical and empirical identification of the sign and magnitudes of the comparative statics and marginal effects.

The model incorporates three features, which allow it to better capture how platforms operate in the new economy.

(i) *Heterogeneity.* Individuals are endowed with both heterogeneous preferences and heterogeneous preferences over heterogeneity (composition effects).

(ii) *Pricing strategies.* Firms incorporate both a size and composition effect into their pricing strategies.

(iii) *Nonpricing strategies.* Firms have nonprice tools at their disposal to strategically leverage their installed base through coordinating expectations and influencing composition (cultivation).

Demographic characteristics of current users of an online dating platform affect the platform's value to a prospective user. In online video gaming, a skilled player of a massively multiplayer online video game receives positive utility when there are many other players online, but receives an extra payoff if many of the other players are skilled as well. The skilled player may discount the value if many of the other players are beginners. Traits valued subjectively, such as competitiveness and sportsmanship, also influence value. Including heterogeneity and composition effects dramatically alters the properties of platforms. A composition effect incentivizes platforms to strategically alter their composition, affecting the pricing statics.

I show that a platform's price is generally nonmonotonic in the size of its installed base. When composition matters, there is no direct relationship between a platform's price and its size, as its composition can have a greater impact on prices than its size. Moreover, changes in size often induce changes in composition. Platforms with identical features can have identically sized installed bases with a significant degree of price dispersion.[4] Market share and market dominance are thus no longer analogous to platform success. Instead,like prices, profitability can vary inversely with market share rather than moving with it. Small platforms can both survive and thrive given the appropriate composition of users. While this is obvious when platform target users vertically (e.g., by income), it is not obvious *a priori* that such an inverse relationship persists under horizontal differentiation. Predicting the effects of changes in installed bases becomes an empirical question rather than a theoretical question. Such an environment reverses our understanding of the dominant-firm fringe-firm paradigm: the fringe can be strategically small to leverage users' composition valuations, while larger platforms rely on size to overcome deficits in composition.

With composition effects, multiple equilibria exist, even with the same market shares. Suppose the market is split evenly between two platforms. If their composition is identical, then they will charge identical prices. Shuffling the consumers around can increase each platform's value through composition without altering the size of each installed base, uniformly raising prices. This point reiterates the idea that market share and traditional measures of "dominance" can be misleading.

Beyond prices, a component of each platform's strategy is to shape the composition of its user base and the value of composition in the population at large, through I focus only on the first. When platforms can price discriminate, they accomplish this task using selection-by-indicators. In many cases, such discrimination is infeasible, so the platforms rely on nonpricing strategies to

[4]This price dispersion is attributable to differences in the composition of the platforms' installed bases, unlike much of the price dispersion literature, which relies on asymmetric or incomplete information. See, for example, Salop and Stiglitz (1977), Varian (1980), Burdett and Judd (1983), Stahl (1989), Sorensen (2000), and Baylis and Perloff (2002). Injecting information or decreasing the costs of information acquisition would not resolve price dispersion driven by composition effects.

influence composition. I call this strategy *cultivation.* Cultivation injects a new feedback loop into the relationship between installed base and prices, directly influencing consumer expectations. Returning to the online dating platforms example, platforms such as ChristianMingle and JDate cultivate to coordinate expectations over religion. The League and Raya cultivate over educational and social elitism (e.g., wealth, education, and celebrity status). In video game markets, Nintendo has cultivated a smaller but more valuable tight-knit network than its competitors (Shankar and Bayus, 2003).

At the outset, platforms can influence consumers' initial purchasing decisions through a costly investment, cultivating across the characteristics consumers care about.[5] Together, composition effects and cultivation are able to explain the presence of multiproduct firms in platform markets. Without composition, a firm cannibalizes its demand by splitting a single platform into multiple platforms unless mediating circumstances such as diseconomies of scale are present. With composition effects, these splits can be value-enhancing, increasing each user's willingness to pay even though the platform is smaller. Cultivation allows for better market segmentation, further enhancing the value of multiple platforms.

Online dating platforms are the quintessential example of a marketplace in which users hold heterogeneous preferences over heterogeneity, platforms are willing and able to cultivate such heterogeneity, and users face identical prices (price discrimination generally occurs only through rate of time preferences). The network value of a dating platform depends on both the number of potential matches and the traits of potential matches. Not all potential matches are valued equally. Hitsch *et al.* (2010b) show that users prefer homogeneity over a fairly large subset of characteristics such as race and religion. A preference for homogeneity along racial lines is also shown in Fisman *et al.* (2008). Hitsch *et al.* (2010a,b) and Fisman *et al.* (2008) collectively show that there are differences between men and women with respect to how various characteristics, such as income and education are valued. Internalizing that both size and composition effects are significant, platforms leverage their networks to increase profits through selective membership and matching algorithms. Hence, heterogeneous preferences over heterogeneity is an empirical regularity that should be incorporated

This paper is not the first to broadly consider heterogeneity and composition, though it is the first

[5]In a dynamic environment, which I discuss in Section **??**, such cultivation creates path dependence, which the firms can leverage by augmenting the dynamics such that the probability of entering a disadvantageous state in the long run becomes vanishingly small. The firms can also use cultivation to either reinforce or eliminate preferences over composition. Thus, cultivation alters many established long run predictions, such as those in Mitchell and Skrzypacz (2006) and Cabral (2011).

to my knowledge to consider horizontal horizontal composition effects within a single side of a platform.[6] This paper is, however, among the first to incorporate composition, pricing, and nonpricing strategies into a single framework. For example, An and Kiefer (1995), Damiano and Li (2007), Chandra and Collard-Wexler (2009), Athey and Ellison (2011), Henkel and Bock (2013), and Marx and Schummer (2021) all incorporate aspects of heterogeneity and pricing strategies. Through compatibility, Katz and Shapiro (1985), Farrell and Saloner (1985, 1986), Markovich (2008), Markovich and Moenius (2009), and Chen *et al.* (2009) incorporate nonpricing strategies. Besides compatibility, the industrial organization literature on platforms has largely ignored nonpricing strategies.[7]

Much of the previous industrial organization literature assumes that network benefits are (i) homogeneous among users and (ii) a function only of the size of the installed base. Exceptions to (i) are found in a few areas. An and Kiefer (1995) and Henkel and Bock (2013) analyze local networks where individuals receive different network benefits. de Palma and Leruth (1996) and Janssen and Mendys-Kamphorst (2007) study network goods where individuals have heterogeneous (vertical) valuations for the networks. Weil (2010), Gomes and Pavan (2011), and White and Weyl (2016) consider heterogeneous (vertical) valuations in two-sided markets of platform competition. This paper considers only single-sided platforms. The current paper, along with Marx and Schummer (2021) present exceptions to both (i) and (ii) though Marx and Schummer (2021) considers only monopolistic two-sided platforms.[8] Larger platforms either always set higher prices or always set lower prices. It is the horizontal composition effects influencing the platform value that introduce the non-monotonicities.[9]

The standard models of platforms, such as Katz and Shapiro (1985), Fudenberg and Tirole (2000), Doganoglu (2003), those presented in the surveys Farrell and Klemperer (2007), Shy (2011), and Cabral (2011), show that absent external forces, e.g., nonlinear pricing regulations à la Laffont *et al.* (1988a,b) or switching costs Chen and Sacks (2025), the feedback loop between prices and

[6] Chandra and Collard-Wexler (2009) and Athey and Ellison (2011) both incorporate heterogeneity on each side of a two-sided market, but composition plays no role within a side. Damiano and Li (2007) considers unidimensional heterogeneity on each side and price discrimination. White and Weyl (2016) develops a monopoly model of two-sided platforms along the lines of Laffont *et al.* (1988a,b), incorporating vertical heterogeneity of users (income). Marx and Schummer (2021) study two-sided monopoly matching markets with unidimensional vertical heterogeneity. Veiga *et al.* (2017) study the problem of platform design under a vertically heterogeneous population, with composition relevant only to the platform designer.

[7] One noteworthy exception is in format selection by radio stations. In two-sided markets, radio stations select a genre / format that appeals to a particular group of listeners, which influences the class of potential advertisers. See, for example, Waldfogel (2003), Sweeting (2010), and Jeziorski (2014).

[8] Although there is a large literature on matching platforms and heterogeneity, this body of work has not integrated platform pricing .See Marx and Schummer (2021) and the citations therein for details.

[9] Exceptions exist in the switching cost literature. See, for example, Chen and Sacks (2025).

consumer expectations vis à vis installed base is monotonic.[10] Larger platforms always set higher prices or always set lower prices.

Though the industrial organization literature has not studied cultivation, religious and identity-based organizations have been observed cultivating their respective populations. Many models within the economics of religion literature have incorporated aspects of heterogeneity and nonpricing strategies, including Iannaccone (1992, 1994), Berman (2000), McBride (2008, 2015), Carvalho, Koyama and Sacks (2017), Carvalho and Sacks (2021), Carvalho and Sacks (2024), and Carvalho, Rubin and Sacks (2024). I apply these principles to the platforms literature and show that, even in the case of private goods—where free-riding is not an issue—similar incentives apply. Unlike other nonpricing strategies in the network and platforms literature such as compatibility, third-party content, and first-party content, cultivation only affects the network externality offered by the platform and not the value of the platform itself.

Similarly, the literature on the provision of local public goods has incorporated heterogeneity (Easterly and Levine, 1997; Alesina *et al.*, 1999; Alesina and La Ferrara, 2000). Hagiu and Spulber (2013) and Veiga *et al.* (2017) incorporate heterogeneity, pricing, and nonpricing strategies. Vertically differentiated users purchase access to the platform, though composition plays no role within a given side of the platform. In Hagiu and Spulber (2013), the platform developer invests in content, which uniformly increases its value to all users and in Veiga *et al.* (2017), a monopoly platform developer chooses the platform characteristics given the distribution of potential users.

The remainder of the paper is structured as follows. Section 2 develops the model. Section 3 presents the main results. Section 4 discusses the implications of the main results. Section 5 briefly introduces some extensions to the model and Section 6 concludes.

2 The Model

This section develops a model of platforms competing in prices. Users are heterogeneous and endowed with heterogeneous preferences over heterogeneity. The platforms are able to cultivate this heterogeneity to their advantage, though such cultivation is costly.

[10] Marx and Schummer (2021), which has two prices (one for each side of the platform) finds that when prices are independently drawn, prices are uniformly decreasing in the size of the platform and when correlated, the price of one side increases while the other decreases.

2.1 Preliminaries

There are two risk-neutral profit-maximizing platforms indexed by $j = A, B$ and n utility-maximizing users, indexed by i with large n. Each platform j produces a single network good (the platform itself) at zero marginal cost. The platform sells access at price p_j. For simplicity I assume users do not multihome. Each user is endowed with a set of traits $y = (y_1, \ldots, y_T) \in Y$. Each $y_\tau \in Y_\tau$ is a trait. Hence, $Y = Y_1 \times \cdots \times Y_T$. There are $\bar{Y} = \prod_\tau |Y_\tau|$ collections of traits in the population and $\hat{Y} = \sum_\tau |Y_\tau|$ total unique traits.

Example 1. *Suppose that $T = 3$. $Y_1 = \{20, 30, 40\}$ is the set of ages, $Y_2 = \{male, female, other\}$ is the set of genders and $Y_3 = \{Christian, Jewish, Muslim, other\}$ is the set of religions. There are $\bar{Y} = 3 \times 3 \times 4 = 36$ types of users and $\hat{Y} = 3 + 3 + 4 = 10$ total traits. A 20 year old female Christian represents one specific profile.*

Many individuals behave identically. All else equal, a 31 year old and 32 year old may display (nearly) identical preferences. To reduce the dimensionality of types, I bin similar sets of traits together. I call these collections of traits *types*. Every user is defined by her T-dimensional type $z_\ell \in Z$ for $\ell = 1, \ldots, L$, where $L = |Z|$. Hence, $L \leq \bar{Y}$. If $y, y' \in z_\ell$ for $y \neq y'$ and some ℓ, then $L < \bar{Y}$: Z partitions Y into L subsets. Without loss of generality, let $n_\ell = |z\ell > 0$. The necessity of distinguishing between traits, trait profiles, and types will become clear in Section 2.2.

Remark 1. *Specifying the model by linking traits to types in this way provides a structured approach to empirically model heterogeneity and composition. Empirical analyses can be conducted at the user (or trait) level. In cases where L is unclear or unknown, unsupervised machine learning methods such as k-means/modes or Gaussian mixture models can be used to partition the $\bar{Y}$ profiles into L bins.*

Each type z_ℓ places a value on the presence of a type z_m possessing each component trait of $y : x_{\ell m y_r} \in \{-1, 0, 1\}$, where 1 represents a desirable trait, 0 a neutral trait, and -1 an undesirable trait.[11] For example, a type may assign a 1 to all types with the same religion and a -1 to all types with a different religion. These valuations create, for each trait ym an $L \times L$ directed graph. These graphs are stacked to build an $L \times L \times \bar{Y}$ tensor X that completely characterizes how each type z_ℓ values the presence of each type z_m that possesses trait y. As not all traits are equally valued, each layer of the tensor is assigned a weight α_{y_r}. The weight signifies the importance of

[11] More general ranges can be used for the $x_{\ell m y_r}$, e.g., $x_{\ell m y_r} \in \mathbb{R}$.

the trait. For example, religion is likely more important than a type's preferred number of pets, so the α associated with religion will be larger than the α associated with pets. The tensor is then collapsed into an $L \times L \times 1$ directed graph H with elements $h_{\ell m}$ given by

$$h_{\ell m} = \sum_{y_r \in \cup_\tau Y_\tau} \alpha_{y_r} x_{\ell m y_r}. \tag{1}$$

If $h_{\ell m} = 0$, then a type z_ℓ user receives neither a premium nor a penalty from the presence of a type z_m user. Positive values convey premiums while negative values convey penalties.

Remark 2. *If the data structure permits, a singular value decomposition can be applied, whereby the data structure G is decomposed such that $G = V'\Sigma U$, where the α are the elements of Σ. Alternatively, when the $h_{\ell m}$ are observed, the α_{y_r} can be estimated using $h_{\ell m} = \sum_{y_r} \alpha_{y_r} x_{\ell m y_r} + \varepsilon_{\ell m}$. To exploit variation within types, ℓ can be measured at the individual level. However, in many applications the $h_{\ell m}$ are unobserved latent values. While the $h_{\ell m}$ may be unobserved, a binary representation*

$$h^*_{\ell m} = \begin{cases} 1 & \text{if } h_{\ell m} > 0 \\ 0 & \text{if } h_{\ell m} \leq 0 \end{cases}$$

is often observed. In online dating, whether or not a specific type chooses to match with another is observed while the underlying latent weight $h_{\ell m}$ is not. Using the discrete $h_{\ell m}$ as the dependent variable, the $\alpha_{y_{r]}}$ can be estimated using MCMC methods such as a Gibbs Sampler with data augmentation. The latent $h_{\ell m}$ are then pulled from the data augmentation step (Albert and Chib, 1993; Vossmeyer, 2014).

Users hold inherent idiosyncratic preferences $\zeta_j \gg 0$ for each platform j. Following Cabral (2011), I assume that ζ_j is sufficiently large to ensure market coverage. Hence, the relevant decision is the relative preference for platform A: $\xi_A = \zeta_A - \zeta_B$, which is distributed according to the CDF $\Phi(\xi)$ with PDF $\phi(\xi)$.

Assumption 1. *(i) $\Phi(\cdot)$ is continuously differentiable. (ii) $\phi(\xi) > 0$ for all ξ. (iii) $\phi(\xi) = \phi(-\xi)$. (iv) $\Phi(\xi)/\phi(\xi)$ is strictly increasing in ξ. (v) $\phi(\xi|z) = \phi(\xi)$.*

Items (i) and (ii) ensure that the demand curves are well behaved and that the platforms' profit functions are quasiconcave, while (iii) ensures that asymmetry occurs only through pricing and market shares. Most continuous distributions satisfy (iv), which is standard in the literature.[12] The last item assumes that the value of a platform itself is not conditional on the user's type.

[12] Items (i)-(iv) mirror Assumption 1 of Cabral (2011, p. 88).

2.2 Platform Behavior

The platforms each make two sequential choices: cultivation decisions followed by pricing decisions. Cultivation is a costly coordination mechanism that operates by altering consumer expectations prior to the pricing subgame. Through costly investment, platforms can target and attract traits, influencing all types possessing those targeted traits. For example, by targeting Jewish singles, a dating platform attracts Jewish males and females across a spectrum of ages and other traits.

The platforms simultaneously and independently set cultivation vectors $\mathbf{c}_j = (c_{j1}, \ldots, c_{j\hat{Y}}) \in [0,1]^{\hat{Y}}$, where c_{jy_r} corresponds to the investment made by platform j in cultivating trait y_r. Let $\mathbf{c} = (\mathbf{c}_A, \mathbf{c}_B)$ denote the collection of cultivation decisions.

Costs take the form $\kappa(\mathbf{c}_j)$, which may consist of both monetary and nonmonetary components.

Assumption 2. *(i)* $\frac{\partial \kappa \mathbf{c}_j}{\partial c_{jy_r}} > 0$ *and* $\frac{\partial^2 \kappa \mathbf{c}_j}{\partial c_{jy_r}^2} > 0$ *for all* y_r *and* $c_{jy_r} \in (0,1)$. *(ii)* $\frac{\partial^2 \kappa \mathbf{c}_j}{\partial c_{jy_r} \partial c_{jy_s}} > 0$ *for* $y_s \neq y_r$. *(iii)* $\lim_{c_{jy_r} \to 1} \frac{\partial \kappa \mathbf{c}_j}{\partial c_{jy_r}} = \infty$ *for all* y_r.

Cultivation operates by influencing consumer expectations via coordination, which is denoted by the superscript 'e'. Prior to cultivation, the consumers' (prior) expectations are given by $q^e_{j\ell}$. Consumers then receive a signal: the observed cultivation vector $\mathbf{c}$. The expectations are augmented by $\mathbf{c}$, yielding post-cultivation expectations $q^e_{j\ell}(\mathbf{c})$. The expected installed base of platform j is then $q^e_j(\mathbf{c}) = \sum_\ell q^e_{j\ell}(\mathbf{c})$. For tractability, I assume that expectations are updated linearly according to

$$q^e_{j\ell}(\mathbf{c}) = \min\left\{ n_\ell, \max\left\{ 0, \left(1 + \frac{\sum_{y_\tau \in \ell} \alpha_{y_\tau} (c_{jy_\tau} - c_{-jy_\tau})}{\sum_{y_\tau \in \ell} \alpha_{y_\tau}} \right) q^e_{j\ell} \right\} \right\} \tag{2}$$

This piece-wise linear representation presents a simple, tractable formulation with which to work. The results all hold for more general functions as well.

At $\mathbf{c} = 0$, $q^e_{j\ell}(\mathbf{c}) = q^e_{j\ell}$. This representation formalizes cultivation as a coordination mechanism. Symmetry in cultivation is imposed across both platforms: efforts are equally effective. Cultivation is also more effective for more important traits (those with a greater α). Denote by $\mathbf{q}^e(\mathbf{c})$ the collection of expectations.

Definition 1. *Define a type* z_ℓ*'s expected valuation of the heterogeneity-weighted network effects provided by platform* j *by*

$$\tilde{q}^e_{j\ell}(\mathbf{c}) = \sum_{m=1}^{L} q^e_{jm}(\mathbf{c}) h_{\ell m}. \tag{3}$$

Remark 3. *While cultivation is at its core a location decision, there are substantial differences between cultivation and the typical Hotelling-style location game. User locations are not primitive, but instead depend on both the distribution of users in the population and their expectations. Similarly, the ideal location choice of the platform depends on both, adding a feedback loop not present in typical location models.*

As in $\mathbf{q}^e(\mathbf{c})$, denote by $\tilde{\mathbf{q}}^e(\mathbf{c})$ the collection of expected heterogeneity-weighted market shares. After observing the cultivation decisions, the platforms simultaneously and independently set their prices.

2.3 User Behavior

Every individual has unit demand and chooses one of the platforms. By purchasing access to platform j, a type z_ℓ user with preferences ζ_j receives expected utility

$$u^e(j, z_\ell) = \zeta_j + v\big(q^e_j(\mathbf{c}), \theta\tilde{q}^e_{j\ell}(\mathbf{c})\big) - p_j, \tag{4}$$

where $v\big(q^e_j(\mathbf{c}), \theta\tilde{q}^e_{j\ell}(\mathbf{c})\big)$ is the value of the network effect of platform j to this individual and $\theta \geq 0$ is the marginal value of composition. If $\theta = 0$, then composition is irrelevant. The relative importance of composition to size is increasing in θ. The realized utility is given by the same functional, substituting the realized values for the expected values.

2.4 Network Effects

The expected (and realized) value of the network associated with each platform depends on two factors: the platform's size and its heterogeneity-weighted network effect.

Assumption 3. *(i) for all $q^e_j(\mathbf{c}) > 0$ and $\theta > 0$, $\frac{\partial v\left(q^e_j(\mathbf{c}), \theta\tilde{q}^e_{j\ell}(\mathbf{c})\right)}{\partial \tilde{q}^e_{j\ell}} > 0$. (ii) For all fixed $\tilde{q}^e(\mathbf{c})$, $\frac{\partial v\left(q^e_j(\mathbf{c}), \theta\tilde{q}^e_{j\ell}(\mathbf{c})\right)}{\partial q^e_j(\mathbf{c})} > 0$. (iii) For fixed $q^e_j(\mathbf{c})$ and $q^e_j(\mathbf{c})' \neq \mathbf{q}^e_j(\mathbf{c})$, $v\big(q^e_j(\mathbf{c}), \theta\tilde{q}^e_{j\ell}(\mathbf{c})\big) - v\big(q^e_j(\mathbf{c})', \theta\tilde{q}^e_{j\ell}(\mathbf{c})'\big)$ is increasing in $\theta(\tilde{q}^e_{j\ell}(\mathbf{c}) - \tilde{q}^e_{j\ell}(\mathbf{c})')$.*

This assumption also carries over to the realized values.

3 Results

I utilize the subgame-perfect Nash equilibrium (SPE) solution concept. In the pricing stage, I find that the well known coordination problem inducing multiple equilibria both persists and is worsened given the additional dimensions to coordinate over. Those equilibria that, upon an increase in size yield a less desirable composition, may fail to satisfy the monotone comparative static property. The set of prices obtainable in equilibrium is significantly wider when $\theta > 0$ than when $\theta = 0$.

In the cultivation stage, I illustrate endogenous product differentiation over the network externality. The product differentiation acts as a form of equilibrium selection. Depending on the nature of heterogeneity and the desirability of composition, cultivation can either soften or strengthen competitive forces. Such competition can lead to higher prices and a less competitive market, lower prices and a more competitive market, or a mixture with increased price dispersion. A platform may opt for a smaller sized network if it corresponds to a more valuable composition, allowing its competitor to grow large, but with a less valuable composition. Both platforms may benefit from such as scenario.

3.1 Pricing Stage

Fix the cultivation profile at $\mathbf{c}$. By (4) and market coverage, for all user expectations $\mathbf{q}^e$, L cutoff values can be defined:

$$\underbrace{\zeta_A - \zeta_B = \xi_A}_{=\omega_\ell} = \underbrace{\big[p_A - v\big(q^e_A(\mathbf{c}), \theta \tilde{q}^e_{A\ell}(\mathbf{c})\big)\big]}_{\substack{\textbf{expected hedonic price}\\ \textbf{of platform } A}} - \underbrace{\big[p_B - v\big(q^e_B(\mathbf{c}), \theta \tilde{q}^e_{B\ell}(\mathbf{c})\big)\big]}_{\substack{\textbf{expected hedonic price}\\ \textbf{of platform } B}},$$

with each cutoff corresponding to the type-z_ℓ user indifferent between platforms A and B.[13] When there is no confusion, I drop the cultivation profile $\mathbf{c}$. Using these cutoff values, platform A's demand is

$$D_A(p_A, p_B, \mathbf{q}^e) = n - \sum_{\ell=1}^{L} n_\ell \Phi(\omega_\ell), \tag{5}$$

and platform B's demand is

$$D_B(p_B, p_A, \mathbf{q}^e). \tag{6}$$

I omit the $\tilde{\mathbf{q}}^e$ from the demand expressions, as the exogenous $h_{\ell m}$ and the expectations $\mathbf{q}^e$ are sufficient in characterizing $\tilde{\mathbf{q}}^e$. Denote by $p^*_A = p_A(\mathbf{q}^e)$, $p^*_B = p_B(\mathbf{q}^e)$, and $\mathbf{p}(\mathbf{q}^e) = (p^*_A, p^*_B)$ the prices in the equilibrium of the subgame induced by the cultivation profile $\mathbf{c}$, where in each equilibrium $\mathbf{q}^e = \mathbf{q}$ (expectations are fulfilled):

$$\begin{aligned} p^*_A &= \arg\max p_A D_A(p_A, p^*_B, \mathbf{q}^e) \\ p^*_B &= \arg\max p_B D_B(p_B, p^*_A, \mathbf{q}^e). \end{aligned} \tag{7}$$

[13]If there are no consumers of type z_ℓ in platform A's installed base, then $\Phi(\omega_\ell) = 1$.

The results of the pricing stage can be interpreted as outcomes of a single-stage game in which there is no cultivation, only composition effects.

Lemma 1. *If* $\mathbf{p}(\mathbf{q}^e)$ *and* $\hat{\mathbf{p}}(\mathbf{q}^e)$ *both satisfy* (7)*, then* $\mathbf{p}(\mathbf{q}^e) = \hat{\mathbf{p}}(\mathbf{q}^e)$*.*

The proof of this and all results are found in the Appendix. Lemma 1 does not imply that there is a unique SPE; rather that there is a unique equilibrium of each subgame induced by a cultivation profile $\mathbf{c}$. Define

$$\omega_\ell^* = \left[p_A - v\big(q_A^e(\mathbf{c}), \theta\tilde{q}_{A\ell}^e(\mathbf{c})\big)\right] - \left[p_B - v\big(q_B^e(\mathbf{c}), \theta\tilde{q}_{B\ell}^e(\mathbf{c})\big)\right]$$

as the cutoff value for a type z_ℓ user in the equilibrium of the subgame.

Lemma 2. *For every set of expectations* $\mathbf{q}^e$*, there exists a representative user endowed with* $\xi_A = \omega_0$*, defined as the individual whose demand corresponds to the representative demand:* $\Phi(\omega_0) = n^{-1}\sum_{\ell=1}^{L} n_\ell \Phi(\omega_\ell)$*.*

This individual possesses traits that are a weighted representation of all individuals in the economy, For convenience, I define this user's expected heterogeneity-weighted network effect as $\tilde{q}_{j0}^e$ and type as z_0, which is interpreted as an $L+1^{th}$ type. Lemma 2 offers an alternative interpretation to the pricing stage where there is only a single type z_0 user on the market, with the remaining users locked in. The static model is then analogous to the stage game of a single-mover overlapping generations framework.

Definition 2. *If* $\tilde{q}_{j0} > 0$*, then network* j *exhibits a heterogeneity premium [*$H+$ *(H-plus)]. If* $\tilde{q}_{j0} < 0$*, then network* j *exhibits a heterogeneity penalty [*$H-$ *(H-minus)]. If* $\tilde{q}_{j0} = 0$*, then network* j *is heterogeneity neutral [*$H0$ *(H-naught)].*

Remark 4. *The complete comparative static of pricing with respect to the installed base is*

$$\underbrace{\frac{\partial p_j(\mathbf{q}^e)}{\partial q_j^e}}_{\textbf{size effect}} + \underbrace{\frac{\partial p_j(\mathbf{q}^e)}{\partial \tilde{q}_{j0}^e}\frac{\partial \tilde{q}_{j0}^e}{\partial q_j^e}}_{\textbf{composition effect}}. \tag{8}$$

This statement follows structurally from the theorem of the maximum. The traditional models of platforms and network effects often only incorporate size effects (H0 networks) and can therefore be interpreted as special cases of this model by either taking $\theta \to 0$ or assuming $H = \mathbf{0}_{L\times L}$.

The models of local network effects, e.g., An and Kiefer (1995), Henkel and Bock (2013), and the citations therein can also be derived as special cases by assuming that local (known) contacts are given zero weight and nonlocal (unknown) contacts are given negative weight (H0 networks), discounting the network effect. Remark 4 is formalized as follows.

Proposition 1. *Under Lemma 2,*

$$\frac{dp_A(\mathbf{q}^e)}{dq^e_{A\ell}} = \frac{1+\frac{\phi'(\omega_0^*)[1-2\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}}{3+\frac{\phi'(\omega_0^*)[1-2\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}} \left[\underbrace{\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial q^e_{A\ell}}}_{\textbf{size effect}} + \underbrace{\theta\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial \tilde{q}^e_{A0}}h_{0\ell}}_{\textbf{composition effect}}\right] \tag{9}$$

$$\frac{dp_B(\mathbf{q}^e)}{dq^e_{A\ell}} = \frac{\frac{\phi'(\omega_0^*)\Phi(\omega_0^*)}{\phi(\omega_0^*)^2}-1}{3+\frac{\phi'(\omega_0^*)[1-2\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}} \left[\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial q^e_{A\ell}} + \theta\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial \tilde{q}^e_{A0}}h_{0\ell}\right]. \tag{10}$$

Moreover, there exists a cutoff value $\theta^*(\mathbf{q}^e)$ such that for all expectations $q^e_A \leq q^e_B$:

(i) $q^e_A \leq q^e_B$ *but* $p_A(\mathbf{q}^e) > p_B(\mathbf{q}^e)$*,*

(ii) *if* $\theta > \theta^*(\mathbf{q}^e)$ *and* $h_{0\ell} < 0$*, then* $\frac{dp_A(\mathbf{q}^e)}{dq^e_{A\ell}} < 0$*.*

Proposition 1 yields two noteworthy implications. First, the effect of a platform's price with respect to a change in the size of its own installed base [equation (9)] cannot be signed. The sign of the effect depends on the type of user being added and the relationship between that user's type and those of the current installed base. Note that

$$\frac{1+\frac{\phi'(\omega_0^*)[1-2\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}}{3+\frac{\phi'(\omega_0^*)[1-2\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}} > 0,$$

as is the size effect and the first term of the composition effect: $\theta\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial \tilde{q}^e_{A0}}$. The sign of $h_{0\ell}$ is either known or estimable (Remark 2), but varies across types. A positive relationship between a platform's price and the size of its installed base is guaranteed only if either $\theta = 0$ or $h_{0\ell} \geq 0$. If $\theta > 0$ and $h_{0\ell} < 0$, then the relationship can be nonmonotonic depending on the current state $\mathbf{q}^e$ and the magnitude of θ. The effect of a change in the installed base is an empirical question.

Second, when composition effects are present, the effect of a change in the size of a platform's installed base can have an indeterminate effect on a competing platform's price. When composition effects are absent, if one platform grows relative to another, then all else equal, the competing platform must compensate by lowering its price. When composition effects are present, if a platform increases its size, but that increased size comes at the cost of a poorer composition, then the relative value of the unchanged competing platform increases. A platform can increase its size but worsen its composition, which can induce a decrease in price while allowing a competing platform to increase its price: price dispersion without asymmetric information, search, or other frictions.

A simple example illustrates the importance of composition effects.

Example 2. *Suppose $\Phi \sim U$ so the first term in (9) and (10) are $\frac{1}{3}$ and $-\frac{1}{3}$, respectively. Platform A has Q users and there are three potential users to add, a type-1 user with $h > 0$, a type-2 user with $h = 0$, and a type-3 use with $h < 0$. Each increases the number of users to $Q+1$; however, adding the type-1 user yields $\frac{dp_A}{dq} > 0$ and $\frac{dp_B}{dq} < 0$, adding the type-2 user yields $\frac{dp_A}{dq} > 0$ and $\frac{dp_B}{dq} < 0$, and adding the type-3 user yields $\frac{dp_A}{dq} < 0$ and $\frac{dp_B}{dq} > 0$ if θ is sufficiently large.*

The differentiability of $\phi(\cdot)$ and $v(q_j, \theta\tilde{q}^e_{j0})$ are not necessary for the patterns described in Proposition 1 to hold, but are necessary for analytically decomposing the size and composition effects.[14] Moreover, (9) and (10) can be derived without invoking Lemma 2. Define

$$\bar{\Phi} = \sum_{m=1}^{L} \frac{n_m}{n}\Phi(\omega_m), \quad \bar{\phi} = \sum_{m=1}^{L} \frac{n_m}{n}\phi(\omega_m), \quad \bar{\phi}' = \sum_{m=1}^{L} \frac{n_m}{n}\phi'(\omega_m).$$

Then (9) and (10) are represented by the solution to

$$\begin{pmatrix} -2 - \frac{\bar{\phi}'(1-\bar{\Phi})}{\bar{\phi}^2} & 1 + \frac{\bar{\phi}'(1-\bar{\Phi})}{\bar{\phi}^2} \\ 1 - \frac{\bar{\phi}'\bar{\Phi}}{\bar{\phi}^2} & -2 + \frac{\bar{\phi}'\bar{\Phi}}{\bar{\phi}^2} \end{pmatrix} \begin{pmatrix} \frac{dp_A(\mathbf{q}^e)}{dq^e_{A\ell}} \\ \frac{dp_B(\mathbf{q}^e)}{dq^e_{A\ell}} \end{pmatrix} = \begin{pmatrix} \frac{\sum_m \frac{n_m}{n}[\phi(\omega_m)\bar{\phi} + \phi'(\omega_m)(1-\bar{\Phi})]\Omega_m}{\bar{\phi}^2} \\ \frac{\sum_m \frac{n_m}{n}[\phi(\omega_m)\bar{\phi} + \phi'(\omega_m)\bar{\Phi}]\Omega_m}{\bar{\phi}^2} \end{pmatrix},$$

where

$$\Omega_m = \frac{\partial v(q^e_A, \theta\tilde{q}^e_{Am})}{\partial q^e_{A\ell}} + \theta\frac{\partial v(q^e_A, \theta\tilde{q}^e_{Am})}{\partial \tilde{q}^e_{Am}} h_{m\ell}.$$

Ω_m is analogous to the composition and size effects in Proposition 1, weighted and aggregated across all types.

The size effect is only sufficient to explain variations in the prices of network goods when there is either no (or a small) change in composition or no effect of composition. It cannot generate a complete comparative static. While the size effect is uniformly positive, the composition effect varies in both sign and magnitude, leading to the indeterminate total effect.[15]

[14] This statement is proven in the proof of Proposition 1.

[15] In dynamic settings, the size effect can be negative, as in Cabral (2011, p. 84) and Fudenberg and Tirole (2000). Nevertheless, a positive composition effect would still imply indeterminacy in this case.

Proposition 1 implicitly assumed that platform B's installed base was unchanged (the addition of a user to the market). It is also worthwhile to augment the analysis to understand the effect of users switching platforms.

Proposition 2. *Under Lemma 2 and assuming a transfer of users form platform B to A,*

$$\frac{dp_A(\mathbf{q}^e)}{dq^e_{A\ell}} = \frac{1+\frac{\phi'(\omega_0^*)[1-2\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}}{3+\frac{\phi'(\omega_0^*)[1-2\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}}\left[\underbrace{\left(\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial q^e_{A\ell}}+\frac{\partial v(q^e_B,\theta\tilde{q}^e_{B0})}{\partial q^e_{B\ell}}\right)}_{\textbf{size effect}}+\underbrace{\theta\left(\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial \tilde{q}^e_{A0}}+\frac{\partial v(q^e_B,\theta\tilde{q}^e_{B0})}{\partial \tilde{q}^e_{B0}}\right)h_{0\ell}}_{\textbf{composition effect}}\right] \tag{11}$$

$$\frac{dp_B(\mathbf{q}^e)}{dq^e_{A\ell}} = \frac{\frac{\phi'(\omega_0^*)\Phi(\omega_0^*)}{\phi(\omega_0^*)^2}-1}{3+\frac{\phi'(\omega_0^*)[1-2\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}}\left[\left(\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial q^e_{A\ell}}+\frac{\partial v(q^e_B,\theta\tilde{q}^e_{B0})}{\partial q^e_{B\ell}}\right)+\theta\left(\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial \tilde{q}^e_{A0}}+\frac{\partial v(q^e_B,\theta\tilde{q}^e_{B0})}{\partial \tilde{q}^e_{B0}}\right)h_{0\ell}\right]. \tag{12}$$

Moreover, there exists a cutoff value $\theta^{**}(\mathbf{q}^e)$ such that for all expectations $q^e_A \leq q^e_B$:

(i) $q^e_A \leq q^e_B$ *but* $p_A(\mathbf{q}^e) > p_B(\mathbf{q}^e)$,

(ii) *if* $\theta > \theta^{**}(\mathbf{q}^e)$ *and* $h_{0\ell} < 0$, *then* $\frac{dp_A(\mathbf{q}^e)}{dq^e_{A\ell}} < 0$.

The relationship between θ^* and θ^{**} cannot be determined without placing further assumptions on v.[16] For a given θ, the size effect is compounded and places upward pressure on the platform's own price and downward pressure on the competing platform's price. An increase in the size of platform A's installed base necessitates a decrease in the size of platform B's installed base. The composition effect is compounded as well under a transfer of users, though this compounding effect can place either upward or downward pressure on price. When $h_{0\ell} > 0$, the composition effect compounds the price effect, as platform A's composition value increases while platform B's decreases. When $h_{0\ell} < 0$ the opposite occurs, putting downward pressure on the price.

The marginal value of composition θ bounds the composition effects. When θ is sufficiently small, the network is approximately $H0$. In $H0$ networks, the size effect will always dominate the composition effect. The monotone-pricing relationship described in Katz and Shapiro (1985), Cabral (2011), and those in Farrell and Klemperer (2007) and Shy (2011) can be expected to hold in network industries such as software and telecommunications. Empirical evidence for monotone pricing in software is found in Gandal (1994) and Brynjolfsson and Kemerer (1996). Yet in many other network industries, such as internet dating and video games, composition effects are strong and can exert significant force over the pricing.[17] When θ is non-negligible, signing the effect of an increase

[16] The relationship between θ^* and θ^{**} is determined by the relative differences between the size and composition effects in Propositions 1 and 2.

[17] For details on the empirical networks literature, see Birke (2009).

in the size of the installed base requires knowledge of the type being added and how the various types interact with one-another.

Proposition 3. *The effects of H and θ on pricing are as follows.*

(i) *If network j is $H+$, then* $\frac{dp_j(\mathbf{q}^e)}{d\theta} > 0$.

(ii) *If network j is $H-$, then* $\frac{dp_j(\mathbf{q}^e)}{d\theta} < 0$.

(iii) *If network j is $H0$, then* $\frac{dp_j(\mathbf{q}^e)}{d\theta} = 0$.

When θ increases, platforms have a greater incentive to positively affect composition in order to cultivate an $H+$ network. If both platforms are able to cultivate an $H+$ network, then both platforms benefit. Fro large enough θ, the composition effect exceeds the size effect and a small increase (or decrease) in market share can lead to large swings, both positive and negative, in price. These are $H+$ and $H-$ networks. Such markets include online dating websites, social networks, and video games. Shankar and Bayus (2003) provides empirical evidence in the video game industry. A change in the size of a platform's installed base need not correspond to a direct change in the price. Nearly every change in market share is accompanied by a corresponding change in composition, except for the rare case in which the proportions of all types remain unchanged. In any case in which the distribution is affected, either compounding or countering composition effects interact with size effects. As a result, (8) is the appropriate measure to use when considering price changes resulting from changes in installed base.

An increase in installed base, when coupled with an increase in the relative heterogeneity premium (or decrease in the heterogeneity penalty), implies compounding effects and monotone price changes. An increase in market share, when coupled with a decrease in the relative heterogeneity premium (or increase in the heterogeneity penalty), implies countering effects, which can lead to lower prices for larger networks.

Let $\pi_j(p(\mathbf{q}^e))$ denote platform j's profits in the equilibrium of the subgame induced by $\mathbf{c}$.

Corollary 1. *For all expectations $q^e_A \leq q^e_B$ with $\tilde{q}^e_{A0} > \tilde{q}^e_{B0}$, there exists a cutoff value $\bar{\theta}(\mathbf{q}^e)$ such that $\pi_A(p_A(\mathbf{q}^e)) > \pi_B(p_B(\mathbf{q}^e))$ if and only if $\theta > \bar{\theta}(\mathbf{q}^e)$.*

Corollary 1 is a direct result of Propositions 1–3. Composition effects can become important enough that the smaller platform with a better composition can set a higher price than a larger competitor. This price differential can be so severe that the smaller platform is also more profitable than its larger competitor.

Proposition 4. *Under Lemma 2 and assuming a transfer of users from platform B to A,*

$$\frac{d\pi_A(\mathbf{p}(\mathbf{q}^e))}{dq^e_{A\ell}} = np_A(\mathbf{q}^e)\phi(\omega_0^*)\left[\left(\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial q^e_{A\ell}} + \frac{\partial v(q^e_B,\theta\tilde{q}^e_{B0})}{\partial q^e_{B\ell}}\right) + \theta\left(\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial \tilde{q}^e_{A0}} + \frac{\partial v(q^e_B,\theta\tilde{q}^e_{B0})}{\partial \tilde{q}^e_{B0}}\right)h_{0\ell}\right] \tag{13}$$

$$\frac{d\pi_B(\mathbf{p}(\mathbf{q}^e))}{dq^e_{A\ell}} = -np_B(\mathbf{q}^e)\phi(\omega_0^*)\left[\left(\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial q^e_{A\ell}} + \frac{\partial v(q^e_B,\theta\tilde{q}^e_{B0})}{\partial q^e_{B\ell}}\right) + \theta\left(\frac{\partial v(q^e_A,\theta\tilde{q}^e_{A0})}{\partial \tilde{q}^e_{A0}} + \frac{\partial v(q^e_B,\theta\tilde{q}^e_{B0})}{\partial \tilde{q}^e_{B0}}\right)h_{0\ell}\right]. \tag{14}$$

Therefore, $\text{sign}\left\{\frac{d\pi_A(\mathbf{p}(\mathbf{q}^e))}{dq^e_{A\ell}}\right\} \neq \text{sign}\left\{\frac{d\pi_B(\mathbf{p}(\mathbf{q}^e))}{dq^e_{A\ell}}\right\}$.

If instead of a transfer from platform B to A, platform B's installed base remains unchanged, then $\frac{\partial v(q^e_B,\theta\tilde{q}^e_{B0})}{\partial q^e_{B\ell}} = \frac{\partial v(q^e_B,\theta\tilde{q}^e_{B\ell})}{\partial q^e_{B0}} = 0$. Like Propositions 1 and 2, the signs of (13) and (14) depend on θ and $h_{0\ell}$. The nonmonotonicities in prices carry over to profits. If a platform adds users that do not mesh well with the existing base, the platform becomes less valuable to its existing users and thus less profitable as it grows. Equation (13) illustrates the potential for a niche construction strategy: by purging users that are not a good fit, the platform can shrink its size but increase its value to existing users, increasing profits.[18] There is no reliance on a vertical measure of differentiation like income, but rather a horizontal measure. Small $H+$ networks with highly favorable compositions can be significantly more valuable to consumers than large $H0$ or $H-$ networks, allowing for niche, boutique, and exclusive platforms to both survive and thrive.

Proposition 5. *The elasticity of demand for platform j is decreasing in both $\tilde{q}^e_j$ and $\tilde{q}^e_j - \tilde{q}^e_{-j}$.*

When delineating the market by type and increasing the value of composition for a given θ, consumers' responsiveness to prices diminishes, implying a less elastic demand (the composition creates a quasi lock-in effect).

Competition for composition opens up an avenue for product differentiation over the network externality. This avenue is the primary mode of competition observed in online dating markets as features become standardized across platforms.[19] Dating platforms such as ChristianMingle, JDate, Farmers Only, Raya, and The League each target a distinct trait over which to differentiate,

[18] Niche construction has been employed by identity-based organizations (who similarly provide platforms for members of the associated identity group) as a mechanism for radicalization (Carvalho and Sacks, 2024).

[19] Most dating platforms employ similar algorithms built upon a singular value decomposition (Klemens, 2006, p. 61).

creating natural market segmentation. Given that these platforms no longer directly compete with one-another, the platforms have greater flexibility in pricing. In practice, competition for the market often emerges within each segment, as multiple politics-based, religious-based, and other platforms have emerged.

3.2 Cultivation Stage

To isolate the effects of cultivation, I make the following assumption regarding expectations.

Assumption 4. *For each type z_ℓ, $q^e_{A\ell} \sim U(0, n_\ell)$. Let $\mathbf{Q}^e$ denote the random variable.*[20]

Each draw is stochastic with a symmetric average. Without cultivation, each platform has an expected $n/2$ users, with the types split evenly across platforms.

Remark 5. *The comparative static of pricing with respect to composition is*

$$\left[\underbrace{\frac{\partial p_j(\mathbf{q}^e(\mathbf{c}))}{\partial \tilde{q}^e_{j0}(\mathbf{c})}}_{\substack{\textbf{direct composition}\\ \textbf{effect}}} + \underbrace{\frac{\partial p_j(\mathbf{q}^e(\mathbf{c}))}{\partial q^e_{j\ell}(\mathbf{c})}\frac{\partial q^e_{j\ell}(\mathbf{c})}{\partial \tilde{q}^e_{j0}(\mathbf{c})}}_{\substack{\textbf{indirect composition}\\ \textbf{effect}}}\right]\frac{\partial \tilde{q}^e_{j\ell}(\mathbf{c})}{\partial c_{jy_r}}. \tag{15}$$

This decomposition is analogous to the pricing stage. It uncouples the direct effect of cultivation on prices through composition and the indirect effect of cultivation on prices through size.

Given $p_A(\mathbf{q}^e(\mathbf{c}))$ and $p_B(\mathbf{q}^e(\mathbf{c}))$, the platforms' objectives are

$$\begin{aligned} &\max_{\mathbf{c}_A} \{p_A(\mathbf{q}^e(\mathbf{c}))[1 - \Phi(\omega^*_0)] - \kappa(\mathbf{c}_A)\} \\ &\max_{\mathbf{c}_B} \{p_B(\mathbf{q}^e(\mathbf{c}))\Phi(\omega^*_0) - \kappa(\mathbf{c}_B)\}, \end{aligned} \tag{16}$$

with ω^*_0 evaluated at $\mathbf{p}(\mathbf{q}^e(\mathbf{c}))$. The arbitrary first-order condition (FOC) with respect to cultivating trait y_r by platform A is given by

$$\begin{aligned} p_A(\mathbf{q}^e(\mathbf{c}^*))\phi(\omega^*_0) \sum_{\ell:\, y_\tau \in \ell} \Bigg[&\frac{\partial v(q^e_A, \theta \tilde{q}^e_{A0})}{\partial q^e_{A\ell}} + \frac{\partial v(q^e_B, \theta \tilde{q}^e_{B0})}{\partial q^e_{B\ell}} \\ &+ \theta \left(\frac{\partial v(q^e_A, \theta \tilde{q}^e_{A0})}{\partial \tilde{q}^e_{A0}} + \frac{\partial v(q^e_B, \theta \tilde{q}^e_{B0})}{\partial \tilde{q}^e_{B0}} \right) h_{0\ell} \Bigg] f(\boldsymbol{\alpha}) = \frac{\partial \kappa(\mathbf{c}^*_A)}{\partial c_{Ay_r}}, \end{aligned}$$

where

$$f(\boldsymbol{\alpha}) = \begin{cases} \frac{\alpha_{y_\tau}}{\sum_{y_s \in \ell} \alpha_{y_s}} & \text{if } q^e_{A\ell}(\mathbf{c}^*) = \left(1 + \frac{\sum_{y_s \in \ell} \alpha_{y_s}(c_{jy_s} - c_{-jy_s})}{\sum_{y_s} \alpha_{y_s}}\right) q^e_{j\ell} \\ 0 & \text{otherwise.} \end{cases}$$

[20] Any truncated marginal distribution centered about $n_\ell/2$ is sufficient. I employ the uniform distribution for simplicity.

Although not the true comparative static, the two effects are visible, augmented by the change in cultivation $f(\boldsymbol{\alpha})$.

Lemma 3. *There exists at least one SPE.*

While existence is guaranteed, uniqueness is not. The SPE cannot be completely characterized without further restrictions on $\mathbf{q}^e$, H, and θ; however, analyzing the above FOC still provides insights.

Proposition 6. *In all SPE* $(\mathbf{c}^*, \mathbf{p}(\mathbf{q}^e(\mathbf{c}))$ *and for all* H *and* $\mathbf{q}^e$*:*

(i) *There exists a* $\underline{\theta}(\mathbf{q}^e)$ *such that* $\mathbf{c}^*_A, c^*_B \neq \mathbf{0}$ *whenever* $\theta \leq \underline{\theta}(\mathbf{q}^e)$.

(ii) $c^*_{jy_r} > 0$ *if and only if*

$$\sum_{\ell:\, y_r \in \ell} \left[\frac{\partial v(q^e_A, \theta \tilde{q}^e_{A0})}{\partial q^e_{A\ell}} + \frac{\partial v(q^e_B, \theta \tilde{q}^e_{B0})}{\partial q^e_{B\ell}} + \theta \left(\frac{\partial v(q^e_A, \theta \tilde{q}^e_{A0})}{\partial \tilde{q}^e_{A0}} + \frac{\partial v(q^e_B, \theta \tilde{q}^e_{B0})}{\partial \tilde{q}^e_{B0}} \right) h_{0\ell} \right] f(\boldsymbol{\alpha}) > 0.$$

(iii) *Suppose that* $\{\ell : y_r \in \ell\} = \{\ell : y_s \in \ell\}$ *and* $c^*_{jy_r}, c^*_{jy_s} > 0$ *for some traits* y_r, y_s*. Then,* $c_{jy_r} > (=)\, c_{jy_s}$ *if and only if* $\alpha_{y_r} > (=)\, \alpha_{y_s}$.

When θ is small, the rewards to efforts aimed at increasing the size of the installed base are significantly more than efforts aimed at increasing the composition efforts. As both platforms are able to employ strategic cultivation, a prisoner's dilemma emerges, much like the typical advertising game. The platforms' efforts counteract one-another, implying the existence of a profile $\mathbf{c}'$ in which the platforms cultivate to a lesser degree, yielding the same distribution: $\mathbf{q}^e(\mathbf{c}') = \mathbf{q}^e(\mathbf{c}^*)$, but with each platform receiving greater profits.

Trait y is cultivated whenever the total marginal effect of cultivating that trait is positive. The total effect, given in statement (ii) of Proposition 6, considers every type ℓ in which trait y is present. The size effect is positive for each ℓ, though the composition effect can take any sign. Summing the net effect across all ℓ affected by y gives the total marginal effect, which when positive implies that trait y is revenue increasing. When negative, it is revenue decreasing and when zero, revenue neutral. In the latter two cases, not cultivating trait y is a best response.

For any $\theta > 0$, those traits with the greater α carry the most weight through $f(\boldsymbol{\alpha})$, which is nondecreasing in that α. If there are two traits that affect the same set of types, then the more dominant trait, defined by the larger α, is cultivated more intensely.

Definition 3. *Trait y_r (weakly) dominates y_s if $\alpha_{y_r} > (\geq)\, \alpha_{y_s}$. Trait y_r is (weakly) dominant if $\alpha_{y_r} > (\geq) \max_{y_s \neq y_r} \alpha_{y_s}$.*

Dominance is unclear across two traits that affect non-identical sets of types, even if those sets have a nonempty intersection.

Regardless of the size effect, specific traits may be cultivated so long as they are revenue increasing. Thus, cultivation always occurs when the size effect is dominant, as any increase in the size of the installed base is valuable. This outcome of Proposition 6 resembles the traditional literature, although the strategy pursued is distinct. While useful, it is more interesting to consider investments increasing the installed base when not all increases are treated equally. Some increases can even harm the platform. Unfortunately, precise characterizations of the SPE are unavailable without further restrictions on H. For the remainder of this section, suppose that θ is sufficiently large so that the composition effect dominates the size effect. In what follows, I consider several specifications of H that resemble real-world occurrences, such as in-group bias, out-group bias, and grouping.

Various restrictions will be placed on the $h_{\ell m}$.

3.2.1 In-Group Bias

In-group bias occurs whenever an individual prefers an own-type to an other-type. This broad definition implies that $h_{\ell\ell} > h_{\ell m} > 0$ for $\ell \neq m$ constitutes in-group bias. I use a more restrictive definition where other-types impose a negative externality, rather than a smaller positive externality to eliminate uninteresting cases.

Definition 4. *In-group bias exists if $h_{\ell\ell} > 0$ for each ℓ and $h_{\ell m} < 0$ for all $m \neq \ell$. The in-group bias is weak if $\sum_{m \neq \ell} |h_{\ell m}| > h_{\ell\ell}$ for all ℓ. The in-group bias is strong if $\sum_{m \neq \ell} |h_{\ell m} < h_{\ell\ell}$ for all ℓ.*

The strength of in-group bias is defined by the diagonal-dominance of H. If H is diagonal dominant, then the bias is strong and if H is diagonal nondominant, then it is weak. Strong bias implies that the presence of a single own-type outweighs the presence of one of each of the other $L - 1$ other types for every type. Weak bias implies the opposite. If the in-group bias is strong, then

$\sum_{m\neq 0} |h_{0m}| > h_{00}$. An analogous relationship holds for weak in-group bias.

Proposition 7. *Suppose that* $\mathbf{q}^e = \mathbb{E}\mathbf{Q}^e$ *and that* θ *is sufficiently large. If, for some nonnegative* ε*,* $\sum_{m\neq \ell} |h_{\ell m}| - h_{\ell\ell} > \varepsilon$*, then there is a unique SPE in which* $\mathbf{c}^* = \mathbf{0}$*. If* $\sum_{m\neq \ell} |h_{\ell m}| - h_{\ell\ell} < -\varepsilon$*, then in every SPE,*

(i) $\mathbf{c}^* \neq \mathbf{0}$,

(ii) $c^*_{jy_r} > 0$ *for at least one* j *and all* y_r*.*

If $\frac{\partial^2 v(q_j^e(\mathbf{c}), \theta\tilde{q}^e_{j\ell})}{\partial q^{e2}_{j\ell}} = 0$, then $\varepsilon = 0$. If $\frac{\partial^2 v(q_j^e(\mathbf{c}), \theta\tilde{q}^e_{j\ell})}{\partial q^{e2}_{j\ell}} > 0$, then $\varepsilon > 0$ and ε is increasing in the steepness of $\frac{\partial v(q_j^e(\mathbf{c}), \theta\tilde{q}^e_{j\ell})}{\partial q^{e}_{j\ell}}$. If $\frac{\partial^2 v(q_j^e(\mathbf{c}), \theta\tilde{q}^e_{j\ell})}{\partial q^{e2}_{j\ell}} < 0$, then weak in-group bias is not necessary for Proposition 7, though it is sufficient. When the in-group bias is sufficiently strong, the additional positive externality of increasing the presence of in-group members outweighs the negative externality induced on existing out-group members, As a result, there is always the incentive to cultivate at least one trait. Moreover, increasing the presence of each type of user is also value-enhancing, so all traits are cultivated, though not necessarily by the same platform. When the in-group bias is sufficiently weak, the additional negative externality outweighs any positive externality generated by size effects and positive externalities from a larger in-group. In this environment, the ideal is a wholly in-group composition, though this is only obtainable under a small subset of conditions on $\mathbf{q}^e$. Under the assumption $\mathbf{q}^e = \mathbb{E}\mathbf{Q}^e$, $\tilde{q}^e_{j\ell} = \frac{n}{2L}\sum_{m=1}^{L} h_{\ell m}$. From the perspective of each z_ℓ user, the platform is $H+$ if the in-group bias is weak, $H-$ if it is strong, and $H0$ if it is balanced.

Under asymmetric expectations, a wide variety of outcomes are possible. When the distribution of a specific type across platforms is significantly unbalanced and H exhibits weak in-group bias, an SPE without cultivation may no longer exist. The platform with the large share of type z_ℓ users seeks to increase their presence. Although the in-group bias is weak, the number of individuals benefiting from the increase of the in-group outweighs the negative externality imposed on those individuals who suffer from the presence of an out-group member. Whether or not the SPE is effected depends on how weak the in-group bias is, i.e., the magnitude of $\sum_{m\neq \ell} |h_{\ell m}| - h_{\ell\ell}$.

Proposition 8. *Suppose* θ *is sufficiently large. For every asymmetric* $\mathbf{q}^e$*, there exists a positive* ε' *such that an SPE without cultivation exists if and only if* $\sum_{m\neq \ell} |h_{\ell m}| - h_{\ell\ell} > \varepsilon'$*.*

When $\sum_{m\neq\ell} |h_{\ell m}| - h_{\ell\ell} > \varepsilon'$, the aggregate negative externality outweighs the aggregate positive externality.

3.2.2 Out-Group Bias

Like in-group bias, I use a stronger than necessary definition of out-group bias: $h_\ell < 0$ for each ℓ and $h_{\ell m} > 0$ for every $m \neq \ell$.

Definition 5. *The out-group bias is weak if $\sum_{m\neq\ell} h_{\ell m} > |h_{\ell\ell}|$ for all ℓ. The out-group bias is strong if $\sum_{m\neq\ell} h_{\ell m} < |h_{\ell\ell}|$ for all ℓ.*

When out-group bias is present, a tension between types emerges. Suppose that there are two types of users, a minority type and a majority type. The minority is satisfied due to the presence of a large out group, while the majority is analogously dissatisfied. Which party;s concerns are more important to the platform, vis-à-vis profits, depends on the strength of the bias. If a type dominates others according to the magnitude of $h_{\ell\ell}$, then its satisfaction is weighted heavier. Under strong out-group bias, the goal is to shrink the presence of the most dominant types, saturating the platform with the various out-groups. Because this minority has the most intense preferences for the out-group, their satisfaction outweighs the dissatisfaction of the remaining types. Since the platform is not a monopoly, its competitor has a similar strategy in mind.

Proposition 9. *Suppose that |theta is sufficiently large, $|h_{11}| > |h_{22}| > \cdots > |h_{LL}|$, and $h_{m\ell} = h'$ for all $m \neq \ell$ and some value $h' > 0$. If the out-group bias is sufficiently strong, then platform j's profits are maximized when, for a given $\mathbf{q}^e$, $q_{j1} < q_{j2} < \cdots < q_{jL}$.*

In the SPE, too much cultivation occurs, again resembling a prisoner's dilemma. Suppose that $q^e_{j\ell} \approx \frac{n_\ell}{2}$ for each ℓ. If a platform does not cultivate, then its competitor has the incentive to cultivate, targeting h_{LL} the heaviest, moving downward sequentially, turning the most dominant types into minorities on the platform. Thus, there is no SPE without cultivation. The above profile cannot be part of an SPE either. The non-cultivating platform can increase its value by targeting the same group, increasing their numbers, which increases value to the more dominant types by decreasing their relative share in the installed base. In the end $\mathbf{q}^e \approx \mathbf{q}^e(\mathbf{c}^*)$ with $\mathbf{c}^* = \mathbf{0}$: a prisoner's dilemma. An analogous argument holds with an asymmetric $\mathbf{c}^*$ when the initial distribution is asymmetric.

3.2.3 Grouping

Grouping is a generalized form of in-group bias. Akin to in-group bias, $h_{\ell\ell} \geq 0$ for each ℓ. Partition the types into K sets, $L_1, L_2, \ldots, L_K$. For each $k = 1, \ldots, K$, all $\ell, m \in L_k$ and all $m' \notin L_k$, $h_{\ell m} > 0$ and $h_{\ell m'} < 0$. H takes on a block-diagonal shape

$$H = \begin{pmatrix} + & + & - & - & - & - \\ + & + & - & - & - & - \\ - & - & + & + & - & - \\ - & - & + & + & - & - \\ - & - & - & - & + & + \\ - & - & - & - & + & + \end{pmatrix}.$$

Unlike in-group bias, the diagonals $h_{\ell\ell}$ need not be positive. The positive and negative values can each have varying weights ($h_{\ell m}$ need not be equal). Such patterns are often found in online dating markets, where strong preferences over race, religion, education, and other traits are held.

The platform's ideal composition depends on θ. For larger θ, each platform will target specific positive groupings, hoping that its competitor will target the alternative group. When $K = 2$, the platforms will cultivate traits corresponding to members of a group as best as possible (as there may be overlaps when $\bar{Y} > L$). If the two groups are somewhat even in size and value, then there is no tension. An SPE emerges in which each platform targets a different group. Which group is targeted by each platform depends on the initial state $\mathbf{q}^e$. If there is a large disparity between the size or value of each group, again a prisoner's dilemma emerges and the platforms compete for the same group.

Definition 6. *A group k is dominant if*

(i) $\min\{h_{m\ell} : m, \ell \in L_k\} - \max_{k' \neq k} \max\{h_{m\ell} : m, \ell \in L_{k'}\} \gg 0,$

(ii) $|\max\{h_{m\ell} : m \in L_k, \ell \notin L_k\}| - |\min_{k' \neq k} \min\{h_{m\ell} : m \in L_{k'}, \ell \in L_k\}| \gg 0.$

Two groups are similar if, for groups k and k',

(iii) $|h_{m\ell}| - |h_{m'\ell'}| \approx 0$ *for all* $m, \ell \in L_k$ *and* $m', \ell' \in L_{k'}$,

(iv) $|h_{m\ell'}| - |h_{m'\ell}| \approx 0$ *for all* $m, \ell \in L_k$ *and* $m', \ell' \in L_{k'}$.

Statement (i) states that the type with the lowest within-group valuation in group k is sufficiently larger than the type with the highest within-group valuation in all other groups. Statement (ii) states that the type most tolerant of the out-group is significantly less tolerant than the least tolerant type in all of the out-groups. Statements (iii) and (iv) are straightforward.

Proposition 10. *Suppose that $K = 2$, $\bar{Y} = L$, $\mathbf{q}^e = \mathbb{E}\mathbf{Q}^e$, and θ is sufficiently large.*

(i) *If the two groups are similar, then there exists a unique SPE in which*

(a) $c^*_{jy_r} > 0$ *if and only if* $c^*_{-jy_r} = 0$,

(b) $c^*_{jy_r} > 0$ *for at least one* j *and all* y_r,

(c) *If* $c^*_{jy_r} > 0$ *and* $c^*_{jy_s} > 0$ *for all* $y_r \neq y_s$, *then* $h_{y_r y_s} > 0$.

(ii) *If group 1 is dominant, then there exists a SPE in which*

(a) $c^*_{jy_r} = c^*_{-jy_r}$ *for all* y_r,

(b) $c^*_{jy_r} > 0$ *for all* j *and* $y_r \in L_1$,

(c) $c^*_{jy_s} > 0$ *for all* j *and all* y_s *such that* $h_{y_r y_s} > 0$ *for all* $y_r \in L_1$,

(d) $c_{jy_t} = 0$ *for all* j *and all* y_t *such that* $h_{y_r y_t} < 0$ *for all* $y_r \in L_1$.

Case (i) formalizes an equilibrium with delineation of the market by group. Each platform cultivates a different group and profits are strictly greater than under a zero-cultivation profile. Case (ii) formalizes the prisoner's dilemma.

When $K > 2$, the delineation of the user base by type is less precise. Without relaxing assumptions on ζ_A and ζ_B eliminating market coverage, cultivation is not as effective.[21] Each platform is forced to have both in- and out-groups.

So far, cultivation alters expectations by increasing the presence of types possessing cultivated traits. Often, other traits and types are affected. I refer to these effects as enhancing or refining.

3.2.4 Enhancing and Refining Effects

Cultivation often has a refining effect: targeting one set of traits negatively affects individuals with with related traits, pushing them away from the platform. For example. cultivating a specific age, education, religion, or political affiliation in online dating. If a dating platform announces itself as a platform for political conservatives to find other political conservatives, both conservatives and liberals will adjust their expectations accordingly. Not expecting to find a match, liberal singles

[21] Without market coverage, platforms will price and cultivate such that only one group joins each platform under case (i).. Under case (ii), both of the platforms will cultivate and split the dominant group.

adjust their expectations downward. When traits are complementary, targeting one set of traits can enhance cultivation, attracting those with the complementary traits.

Incorporating enhancing and refining effects to cultivation increases its efficacy. In the case of refinement, the size of the non-cultivated installed base shrinks, but this shrinkage increases the value of the composition effect. For large θ, this shrinkage is a net positive for the platform. A platform targeting a small coalition with strong in-group bias benefits from refinement. It shrinks its installed base so that in the limit only members of the coalition join the platform. This coalition is not valuable in the "vertical" sense; that is, they do not possess a higher income level or a higher willingness to pay for the platform in general. Rather, the value arises horizontally through homogeneity. The platform's exclusivity is driving its value. It is not exclusive in the typical sense of niche or luxury goods, but exclusive to a specific group with preferences to remain isolated—were that group to increase in size, the platform and users would attract them and all benefit. In this context, cultivation acts as a public good where each platform prefers the other to cultivate and through refinement, receive a share of the benefits. In equilibrium, one platform acquiesces and cultivates.

The market resembles the dominant firm-fringe firm paradigm; however, the fringe platform is not "fringe" in the typical sense. Rather, the fringe (cultivating) platform is strategically small by its own doing and the non-cultivating platform's "dominant" position (in terms of market share) is only so as the rational response to its cultivating counterpart. It is driven not by the larger platform, but by the smaller platform. Both platforms may be more profitable than they would be if they split the market evenly. The cultivating platform has fewer but more valuable (in terms of network externalities) adopters while the larger platform is compensated by the increasing returns generated by more users.

A dating platform such as JDate faces such a trade-off. Jewish individuals make up approximately 2.4% of the adult population in the United States (Pew Research Center, 2021). Proposition 10(i) shows how it can be highly profitable to considate the network and leveerage composition effects into prices that match its much larger competitors. JDate, as of May 2025, charges a single month price of \$59.99 (Kottemann, 2025) while Match.com's base price is \$31.99 for a single month (WeGoDating, 2025). JDate has only a fraction of the installed base of Match—2 million (Belz, 2024) compared to 5.5 million (Curry, 2025).

4 Composition: Collusion and Competition

The model and analysis of Sections 2 and 3 are also useful in understanding environments outside of the direct scope of the model. I emphasize two such environments. First, identifying collusion via price fixing can be difficult when the goods in question possess network externalities. This issue is exacerbated by preferences over composition. Ignoring composition effects leads to false positives by attributing higher than expected (or lower than expected) prices to factors outside of market competition. Second, it is difficult to explain a multiproduct firm in a network industry when the products are similar. Without regards to composition, segmenting users into multiple platforms cannibalizes demand. By incorporating composition effects, such separation can be demand-enhancing rather than cannibalizing. The following subsections formally illustrate each of these ideas.

4.1 Identifying Collusion

If composition is ignored and consumers are cultivated so that size remains constant but the composition is improved, then equilibrium prices will appear to be higher than what comeptition supports. In reality, prices are still the result of non-cooperative strategic interactions. To formally illustrate this point, suppose that (i) $\bar{Y} = L$, (ii) $n_\ell = \frac{n}{L}$, (iii) $q^e_{j\ell} = \frac{n_\ell}{2}$ for each ℓ and j, and (iv) $\theta \gg 0$. For simplicity, I also assume that $|h_{\ell m}| = h^* > 0$ is for all ℓ, m. H takes on the block diagonal shape

$$H = \begin{pmatrix} \mathbf{h}^* & -\mathbf{h}^* \\ -\mathbf{h}^* & \mathbf{h}^* \end{pmatrix}, \quad \text{where } \mathbf{h}^* = \begin{pmatrix} h^* & \cdots & h^* \\ \vdots & \ddots & \vdots \\ h^* & \cdots & h^* \end{pmatrix}.$$

By Proposition 10, an SPE exists in which all types are cultivated but no two types are cultivated by the same platform. In this equilibrium, the platforms split the market evenly. By the symmetry assumptions of the user base, $p^*_A = p^*_B$. Setting $\theta = 0$ and $\mathbf{c} = \mathbf{0}$ replicates a model without composition effects with prices $p'_A = p'_B < p^*_A = p^*_B$. Thus, if prices p^*_A and p^*_B are observed but a model without composition effects predicts that competition would yield prices p'_A and p'_B, then the platforms could face unwarranted scrutiny for collusive behavior when prices do in fact reflect the noncooperative environment.

Interestingly, prices can also be lower than what would be predicted by a model without composition effects. If the platforms are $H-$, then each platform sets a lower price than if they were $H0$ or $H+$. Combining these lower than expected prices with high startup or fixed costs may lead to false positives in the other direction. Estimating prices without accounting for composition effects, but observing prices that correspond to the SPE leads to predicting predatory pricing when no such pricing is occurring.

4.2 Singular Ownership of Multiple Platforms

Match Group Inc. operates not only its Match website, but other platforms such as Democratic People Meet, Republican People Meet, The League, Hinge, and Tinder. A single entity owning and operating multiple platforms is not unique to the online dating industry, for example PayPal and Venmo are owned by PayPal Holdings. Nonetheless, these examples illustrate the importance of accounting for composition effects when considering network goods.

Composition effects and strong preferences over types can readily explain why users may prefer multiple independent platforms over a single unified platform. By segregating users by type, each smaller platform becomes increasingly valuable to its base. The gains from the increased positive composition effect outweighs the loss from the decreased size effect. These exclusive platforms are than priced higher than the original larger platform, strictly increasing profits (assuming the additional costs of operating multiple platforms are not too large).

With slight changes to the model, these points can be formally illustrated.

4.2.1 Formal Analysis

Instead of two platforms competing, suppose that there is a single owner that operates up to $J \geq 1$ platforms. Also suppose that ξ_A is constant across all users, i.e., $\phi(\xi_A)$ is a single point mass. This simplifies the analysis so that when operating a single platform, profit maximization entails capturing all n users when there are no composition effects. Cultivation is still a potentially profitable strategy, though any SPE from Section 3 in which a prisoner's dilemma emerges is immediately eliminated with a multiproduct monopolist. Th rest of the model is as described in Section 2.

Proposition 11. *For θ sufficiently small, the owner maximizes profits by operating a single platform.*

Suppose $\theta = 0$. In this case, dividing a single large platform into smaller, independent platforms lowers $v(n, 0)$ to $v(q, 0) < v(n, 0)$ for every $q < n$ and all users. When there is a single platform, the profit maximizing price is given by $p^* = \xi_A + v(n, 0)$, with profits np^*. For multiple platforms, each platform j has a price of $p_j = \xi_A + v(q_j, 0) < p_j$ and profits $q_j p_j$. Since $\sum_{q_j} = n$ and each $p_j < p^*$, $\sum_j q_j p_j < np^*$.

As θ grows larger, a more profitable opportunity arises when consumers hold specific preferences over composition.

Proposition 12. *For θ sufficiently large and either in-group bias, out-group bias, or grouping, the owner maximizes profits by operating at least two platforms.*

More generally, any H that exhibits in-group bias, out-group bias, or grouping on a subset of H satisfies the proposition.

Proposition 12 can also explain why search filters, like those on dating platforms, do not represent a perfect substitute to operating multiple platforms. These search algorithms decrease the cost of matching (increasing the value of the network effect); however, the valuation of the platform also depends on the presence of individuals for which there may not be a direct interaction.

Example 3. *A man in his early 30s with children faces competition in the dating market from other men, both of the same and different types (e.g., a man in his early 30s without children. Similarly, women face competition from women of the same and different types.*

These search algorithms cannot prevent an individual's desired type from interacting with the "competition." Separation of the platforms does mitigate this issue. This idea extends well beyond dating markets to platforms broadly.

5 Multi-Sided Platforms and Dynamic Considerations

Many of the results of Section 3 can be applied to more dynamic settings as well as to multi-sided platforms (e.g., advertisers and end-users on a social network). In what follows, I provide some intuition on some of these extensions with a focus on how composition and cultivate affect these environments. A complete formal analysis is left for future work.

5.1 Multi-Sided Platforms

Many modern platforms are multi-sided. End users do not pay for the service. They act as loss leaders while advertisers pay to reach users through the platform. It merits questioning if advertisers prefer, as defined by willingness to pay for access, a smaller platform.[22] Naturally, advertisers of niche, status, or luxury goods would to prevent over-saturation; however, these goods are often endowed with negative network externalities (such as the snob effect). Interestingly, an argument exists for advertisers selling more typical goods to prefer smaller platforms with a more appropriate composition.

Suppose a product targets a specific demographic. There are two platforms: one with 10,000 users, all of whom are members of the target demographic and one with 50,000 users, 5,000 of whom are from the target demographic. In this case, the advertiser is willing to pay more for the smaller platform. Yet, if the second platform is augmented so that there are still 50,000 in total users, with 11,000 from the target demographic, then the advertiser may still be willing to pay more for the smaller platform. While seemingly counter-intuitive, composition effects and the above analysis provides evidence to support this claim.

On the larger platform, advertisers compete for user attention with other advertisers. This competition includes advertisers targeting the specified demographic, advertisers targeting other demographics, and those targeting the general population. As a result, the likelihood of reaching the desired users is decreased, particularly by those advertisers targeting the general population.[23] Given a lower expected effectiveness, a lower willingness to pay follows.

In the context of social networks, advertisers face a similar issue, even without the presence of other advertisers. There is competition for views from other users. On larger platforms with many posts, the advertisements become buried and suffer from similar clutter effects as described in Ha and McCann (2008). Again, the result is a higher willingness to pay for the smaller platform with the greater likelihood of reaching the demographic of interest.

5.2 Dynamic Composition Effects

Section 3 illustrated how prices and profits are affected by composition effects and the valuation of composition θ. Platforms are also able to leverage composition through cultivation to increase prices and profits. As θ increases, so too do the benefits of cultivation. This relationship introduces a potential dynamically profitable opportunity for platforms: influencing the value of composition. If a platform can both cultivate a valuable installed base *and* increase how consumers value composition via θ, then over time, both prices and profits rise.

[22] See Chandra and Collard-Wexler (2009) for a study of advertiser behavior under mergers.

[23] Evidence in support of this claim is found in Cho and Cheon (2004) and Ha and McCann (2008).

Strategies similar to influencing θ have been utilized by identity groups, such as religious organizations. Carvalho and Sacks (2024) studies the dynamics of endogenous discrimination and niche construction, where leaders of identity groups leverage the environment to increase payoffs. Platforms beyond the context of identity-based organizations can employ similar strategies. Early investments in cultivation and increasing θ can yield significant long run dividends. Moreover, if cultivation efforts need only be temporary (e.g., via lock-in), then the long run returns are even greater.

If θ can be increased by platform activities, then it is similarly plausible that a lack of investment can lead θ to decay. As θ shrinks, composition becomes less important and platforms will invest less in cultivating composition. In the long run, the dynamics converge to those in Cabral (2011).

6 Concluding Remarks

Prior to this paper, composition effects have been largely absent from the study of network goods and platforms, and relatively absent from the general literature on market externalities.[24] I have shown that many of the equilibrium and comparative static properties of price competition in the presence of network externalities need not hold when incorporating heterogeneous preferences over heterogeneity.

In particular, a platform's price is no longer necessarily monotonic in its market share, which implies that a platform's market share is no longer a sufficient statistic for its success. The proper comparative static must incorporate the direct size effects and the indirect composition effects. The model analytically decomposes the comparative static to isolate these effects. The importance of composition effects opens up a new area of study within the realm of imperfect competition: the cultivation of heterogeneity and product differentiation over the externality. I have sown that platforms can use cultivation to coordinate expectations and endogenously influence the valuations in both the population of the platform's adopters and the population at large. The effects of cultivation can be similarly studied by decomposing the pricing effects into a direct effect though composition and an indirect effect through the installed base. Although I linked many of the

[24] Again, exceptions include the study of local public goods and some work on club goods. In the provision of local public goods, heterogeneity among consumers, when couple with preferences over heterogeneity, has substantial impacts. Individuals' willingness to provide local public goods through taxation decreases as the degree of ethnic fragmentation in the local population increases (Easterly and Levine, 1997; Alesina *et al.*, 1999; Alesina and La Ferrara, 2000). This finding indicates that individual attachment to goods with consumption externalities is directly affected by the composition of its users.

assumptions and results to dating platforms, the results are more general and can be applied to many networks found in the new economy.

Appendix

Proof of Lemma 1.

Proof. For a given cultivation profile $\mathbf{c}$, the platform's objective functions are

$$\max_{p_A} p_A \left[n - \sum_{\ell=1}^{L} n_\ell \Phi(\omega_\ell) \right]$$
$$\max_{p_B} p_B \sum_{\ell=1}^{L} n_\ell \Phi(\omega_\ell).$$

The corresponding first-order conditions (FOC) are

$$p_A = \frac{n - \sum_{\ell=1}^{L} n_\ell \Phi(\omega_\ell)}{\sum_{\ell=1}^{L} n_\ell \phi(\omega_\ell)} \tag{17}$$
$$p_B = \frac{\sum_{\ell=1}^{L} n_\ell \Phi(\omega_\ell)}{\sum_{\ell=1}^{L} n_\ell \phi(\omega_\ell)}. \tag{18}$$

Multiplying the right-hand side by n^{-1}/n^{-1} yields

$$p_A = \frac{1 - \sum_{\ell=1}^{L} \frac{n_\ell}{n} \Phi(\omega_\ell)}{\sum_{\ell=1}^{L} \frac{n_\ell}{n} \phi(\omega_\ell)}$$
$$p_B = \frac{\sum_{\ell=1}^{L} \frac{n_\ell}{n} \Phi(\omega_\ell)}{\sum_{\ell=1}^{L} \frac{n_\ell}{n} \phi(\omega_\ell)}.$$

Invoking Lemma 2, I substitute for the representative consumer:

$$p_A = \frac{1 - \Phi(\omega_0)}{\phi(\omega_0)} \tag{19}$$
$$p_B = \frac{\Phi(\omega_0)}{\phi(\omega_0)}. \tag{20}$$

Subtracting (20) from (19) yields

$$p_A - p_B = \frac{1 - 2\Phi(\omega_0)}{\phi(\omega_0)}. \tag{21}$$

By Assumption 1(iv), the right-hand side of (21) is bounded and strictly decreasing in p_A while the left-hand side is unbounded and strictly increasing in p_A. Hence, for any $\mathbf{q}^e$, there is a unique $p_A - p_B$ satisfying (21). Given $p_A - p_B$, (19) and (20), a unique p_A and p_B can be backed out for each $\mathbf{q}^e$. □

Proof of Lemma 2.

Proof. The existence of a representative individual with cutoff ω_0 follows from two points. First, $\Phi(\omega_\ell) \leq 1$ for all ℓ and $\sum_{\ell=1}^{L} n_\ell = n$, which implies that

$$\sum_{\ell=1}^{L} n_\ell \Phi(\omega_\ell) \leq n \Longrightarrow \sum_{\ell=1}^{L} \frac{n_\ell}{n} \Phi(\omega_\ell) \leq 1.$$

Second, bu the continuity of $\Phi(\cdot)$, for every $\sum_{\ell=1}^{L} \frac{n_\ell}{n} \Phi(\omega_\ell) = \Omega \in [0,1]$, there must exist a value ω_0 such that

$$\Omega = \sum_{\ell=1}^{L} \frac{n_\ell}{n} \Phi(\omega_\ell) = \Phi(\omega_0),$$

completing the proof. □

Proofs of Propositions 1 and 2.

Proof. I jointly prove Propositions 1 and 2. From (19) and (20),

$$\begin{aligned} 0 &= -p_A(\mathbf{q}^e) + \frac{1-\Phi(\omega_0^*)}{\phi(\omega_0^*)} \\ 0 &= -p_B(\mathbf{q}^e) + \frac{\Phi(\omega_0^*)}{\phi(\omega_0^*)}. \end{aligned}$$

Totally differentiating each (and setting those differentials not of interest to zero) yields

$$\begin{aligned} 0 = -dp_A(\mathbf{q}^e) - \frac{1}{\phi(\omega_0^*)^2}\left(\phi(\omega_0^*)^2 + \phi'(\omega_0^*)[1-\Phi(\omega_0^*)]\right)\Bigg[dp_A(\mathbf{q}^e) - dp_B(\mathbf{q}^e) - \left(\frac{\partial v(q_A^e, \theta\tilde{q}_{A0}^e)}{\partial q_{A\ell}^e}\right. \\ \left. + \theta \frac{\partial v(q_A^e, \theta\tilde{q}_{A0}^e)}{\partial \tilde{q}_{A0}^e}\frac{\partial \tilde{q}_{A0}^e}{\partial q_{A\ell}^e}\right) dq_{A\ell}^e + \left(\frac{\partial v(q_B^e, \theta\tilde{q}_{B0}^e)}{\partial q_{B\ell}^e}\frac{\partial q_{B\ell}^e}{\partial q_{A\ell}^e} + \theta \frac{\partial v(q_B^e, \theta\tilde{q}_{B0}^e)}{\partial \tilde{q}_{B0}^e}\frac{\partial \tilde{q}_{B0}^e}{\partial q_{A\ell}^e}\right) dq_{A\ell}^e\Bigg] \end{aligned}$$

$$\begin{aligned} 0 = -dp_B(\mathbf{q}^e) - \frac{1}{\phi(\omega_0^*)^2}\left(\phi(\omega_0^*)^2 - \phi'(\omega_0^*)\Phi(\omega_0^*)\right)\Bigg[dp_A(\mathbf{q}^e) - dp_B(\mathbf{q}^e) - \left(\frac{\partial v(q_A^e, \theta\tilde{q}_{A0}^e)}{\partial q_{A\ell}^e}\right. \\ \left. + \theta \frac{\partial v(q_A^e, \theta\tilde{q}_{A0}^e)}{\partial \tilde{q}_{A0}^e}\frac{\partial \tilde{q}_{A0}^e}{\partial q_{A\ell}^e}\right) dq_{A\ell}^e + \left(\frac{\partial v(q_B^e, \theta\tilde{q}_{B0}^e)}{\partial q_{B\ell}^e}\frac{\partial q_{B\ell}^e}{\partial q_{A\ell}^e} + \theta \frac{\partial v(q_B^e, \theta\tilde{q}_{B0}^e)}{\partial \tilde{q}_{B0}^e}\frac{\partial \tilde{q}_{B0}^e}{\partial q_{A\ell}^e}\right) dq_{A\ell}^e\Bigg]. \end{aligned}$$

As $q_{B\ell}^e = n_\ell - q_{A\ell}^e$ for all ℓ, $\frac{\partial q_B^e}{\partial q_{A\ell}^e} = -1$, and $\frac{\partial \tilde{q}_{B0}^e}{\partial q_{A\ell}^e} = -h_{0\ell}$. Making these substitutions, multiplying both sides of each by $\frac{1}{dq_{A\ell}^e}$, and grouping like terms yields

$$\begin{aligned} 0 = -\left(2 + \frac{\phi'(\omega_0^*)[1-\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}\right)\frac{dp_A(\mathbf{q}^e)}{dq_{A\ell}^e} + \left(1 + \frac{\phi'(\omega_0^*)[1-\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}\right)\frac{dp_B(\mathbf{q}^e)}{dq_{A\ell}^e} \\ + \left(1 + \frac{\phi'(\omega_0^*)[1-\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}\right)\underbrace{\left[\frac{\partial v(q_A^e, \theta\tilde{q}_{A0}^e)}{\partial q_{A\ell}^e} + \frac{\partial v(q_B^e, \theta\tilde{q}_{B0}^e)}{\partial q_{B\ell}^e} + \theta\left(\frac{\partial v(q_A^e, \theta\tilde{q}_{A0}^e)}{\partial \tilde{q}_{A\ell}^e} + \frac{\partial v(q_B^e, \theta\tilde{q}_{B0}^e)}{\partial \tilde{q}_{B\ell}^e}\right)h_{0\ell}\right]}_{=\mu} \end{aligned}$$

$$0 = -\left(1 - \frac{\phi'(\omega_0^*)\Phi(\omega_0^*)}{\phi(\omega_0^*)^2}\right)\frac{dp_A(\mathbf{q}^e)}{dq^e_{A\ell}} + \left(1 + \frac{\phi'(\omega_0^*)[1-\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2}\right)\frac{dp_B(\mathbf{q}^e)}{dq^e_{A\ell}} - \left(1 - \frac{\phi'(\omega_0^*)\Phi(\omega_0^*)}{\phi(\omega_0^*)^2}\right)\mu.$$

The system can be rewritten in matrix form as

$$\underbrace{\begin{pmatrix} -2 - \frac{\phi'(\omega_0^*)[1-\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2} & 1 + \frac{\phi'(\omega_0^*)[1-\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2} \\ 1 - \frac{\phi'(\omega_0^*)\Phi(\omega_0^*)}{\phi(\omega_0^*)^2} & -2 + \frac{\phi'(\omega_0^*)\Phi(\omega_0^*)}{\phi(\omega_0^*)^2} \end{pmatrix}}_{=M} \begin{pmatrix} \frac{dp_A(\mathbf{q}^e)}{dq^e_{A\ell}} \\ \frac{dp_B(\mathbf{q}^e)}{dq^e_{A\ell}} \end{pmatrix} = \begin{pmatrix} -1 - \frac{\phi'(\omega_0^*)[1-\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2} \\ 1 - \frac{\phi'(\omega_0^*)\Phi(\omega_0^*)}{\phi(\omega_0^*)^2} \end{pmatrix} \mu.$$

Note that

$$\det(M) = 3 + \frac{\phi'(\omega_0^*)[1 - 2\Phi(\omega_0^*)]}{\phi(\omega_0^*)^2} \neq 0,$$

so the system is solvable. Moreover, $\phi'(\Omega_0^*) < 0$ if and only if $\Phi(\omega_0^*) > \frac{1}{2}$, so $\det(M) > 0$. The comparative statics in Proposition 2 follow. For Proposition 1, the comparative statics are similarly derived, but setting changes in platform B's installed base to zero.

I prove the monotonicity and nonmonotonicity of the comparative statics without assuming differentiability for a more general result. First I illustrate monotonicity. Without loss of generality, consider expectations $q_A^e \leq q_B^e$. Then, $\omega_0^* \geq \Phi^{-1}\left(\frac{1}{2}\right)$. By Assumption 1(ii) and (iii), $\Phi^{-1}\left(\frac{1}{2}\right) = 0$. Therefore,

$$p_A(\mathbf{q}^e) - p_B(\mathbf{q}^e) \geq v(q_A^e, \theta\tilde{q}^e_{A0}) - v(q_B^e, \theta\tilde{q}^e_{B0}). \tag{22}$$

Now suppose that expectations are such that $\tilde{q}^e_{A0} \geq \tilde{q}^e_{B0}$. By Assumption 3(iii), there exists a θ' such that for all $\theta \geq \theta'$ the right-hand side of (22) is nonnegative, which implies that $p_A(\mathbf{q}^e) - p_B(\mathbf{q}^e) \geq 0$.

To prove nonmonotonicity, define $\bar{q} = \max\{\tilde{q}^e_{A0}, \tilde{q}^e_{B0}\}$. Now suppose that q_j^e increases by one. Let $\eta_j < 0$ and $\eta_{-j} > 0$ denote the respective changes to $\tilde{q}^e_{j0}$ and $\tilde{q}^e_{-j0}$, where by construction $\eta_j < 0$ and $\eta_{-j} > 0$ if and only if $h_{0\ell} < 0$ (when $h_{0\ell} = 0$, $\eta_j = \eta_{-j} = 0$). It is sufficient that the values of the network effects move nonmonotonically with market share:

$$v\left(q_j^e + 1, \theta(\tilde{q}^e_{j0} + \eta_j)\right) \leq v(q_j^e, \theta\tilde{q}^e_{j0})$$
$$v\left(q_{-j}^e - 1, \theta(\tilde{q}^e_{-j0} + \eta_{-j})\right) \geq v(q_{-j}^e, \theta\tilde{q}^e_{-j0}).$$

Define θ'" as the minimum θ that satisfies both inequalities. It follows that the inequalities are also satisfied for all $\theta \geq \theta''$. Setting $\theta^*(\mathbf{q}^e) = \max\{\theta', \theta''\}$ in the case of Proposition 1 and $\theta^{**}(\mathbf{q}^e) = \max\{\theta', \theta''\}$ in the case of Proposition 2 completes the proof. $\square$

Proof of Proposition 3.

Proof. For any fixed set of expectations $\mathbf{q}^e$, a change in θ requires that the relative hedonic price of good A for the representative consumer

$$\omega_0^* = [p_A(\mathbf{q}^e) - v(q_A, \theta\tilde{q}_{A0}^e)] - [p_B(\mathbf{q}^e) - v(q_B^e, \theta\tilde{q}_{B0}^e)]$$

remain constant. Otherwise, the marginal consumer moves and expectations are no longer fulfilled. Moreover, platform j's expected hedonic price, $p_j(\mathbf{q}) - v(q_j^e, \theta\tilde{q}_{j0}^e)$ cannot decrease given a change in θ. Otherwise, $p_j(\mathbf{q}^e)$ is not a maximizer, as it would imply the existence of a higher price p_j' that satisfies the equilibrium conditions.

Now, suppose that both platform A's network and platform B's network are $H+$. If θ increases, then both $v(q_A^e, \theta\tilde{q}_{A0}^e)$ and $v(q_B^e, \theta\tilde{q}_{B0}^e)$ increase, so to keep the hedonic prices and relative hedonic prices constant, $p_A(\mathbf{q}^e)$ and $p_B(\mathbf{q}^e)$ must both increase. By an analogous argument, if θ increases but both platforms' networks are $H-$, both prices must decrease.

Next, suppose that platform A's network is $H+$ while platform B's network is $H-$. Then, it follows that $p_A(\mathbf{q}^e)$ is increasing while $p_B(\mathbf{q}^e)$ is decreasing. If a platform is $H0$, then it follows that its price is unaffected by θ. $\square$

Proof of Corollary 1.

Proof. Corollary 1 follows from Propositions 1–3. Consider expectations $q_A^e < q_B^e$ with $\tilde{q}_{A0}^e > \tilde{q}_{B0}^e$. By Lemma 2 and Assumption 3(iii), $p_A - p_B$ is strictly increasing in θ. Therefore, there exists a $\bar{\theta}(\mathbf{q}^e)$ such that if $\theta > \bar{\theta}(\mathbf{q}^e)$, then in the equilibrium of the subgame, $p_A(\mathbf{q}^e)q_A^e > p_B(\mathbf{q}^e)q_B^e$. $\square$

Proof of Proposition 4.

Proof. Proposition 4 follows from an envelope theorem argument. Fix expectations at $\mathbf{q}^e$. The platforms' objectives are given by

$$np_A[1 - \Phi(\omega_0)]$$

$$np_B\Phi(\omega_0).$$

Differentiating each objective with respect to q^e_{A0} and evaluating at $\mathbf{p}(\mathbf{q}^e)$ yields the expressions $np_A(\mathbf{q}^e)\phi(\omega_0^*)\mu$ and $-np_B(\mathbf{q}^e)\phi(\omega_0^*)\mu$, respectively, where μ is defined in the proof of Propositions 1 and 2. The remainder of the proof follows immediately. $\square$

Proof of Proposition 5.

Proof. Without loss of generality, consider platform B. Recall that

$$D_B(p_B, p_A, \mathbf{q}^e) = \sum_{\ell=1}^{L} n_\ell \Phi(\omega_\ell)$$
$$\implies \frac{1}{n} D_B(p_B, p_A, \mathbf{q}^e) = \sum_{\ell=1}^{L} \frac{n_\ell}{n} \Phi(\omega_\ell),$$

which by Lemma 2, can be rewritten as $D_B(p_B, p_A, \mathbf{q}^e) = \Phi(\omega_0)$. Therefore,

$$\frac{\partial D_B(p_B, p_A, \mathbf{q}^e)}{\partial p_B} = -n\phi(\omega_0),$$

so platform B's elasticity is given by

$$E_B = -np_B \left(\frac{\Phi(\omega_0)}{\phi(\omega_0)} \right)^{-1}.$$

By Assumption 1(iv), $(\Phi(\omega_0)/\phi(\omega_0))^{-1}$ is strictly decreasing in ω_0. As $v(q^e_B, \theta\tilde{q}^e_{B0})$ is strictly increasing in $\tilde{q}^e_{B0}$ and $v(q^e_A, \theta\tilde{q}^e_{A0})$ is strictly decreasing in $\tilde{q}^e_{B0}$, ω_0 is increasing in $\tilde{q}^e_{B0}$ and $\tilde{q}^e_{B0} - \tilde{q}^e_{A0}$, completing the proof. $\square$

Proof of Lemma 3.

Proof. Recall the objectives in (16). At $p_A = p^*_A$ and $p_B = p^*_B$, each objective in (16) is upper semicontinuous. Given that $\mathbf{c}$ is defined over a compact space, a solution exists. $\square$

Proof of Proposition 6.

Proof. Without loss of generality, consider platofrm A. The FOC with respect to c_{Ay_r} is given by

$$p_A(\mathbf{q}^e(\mathbf{c}^*))\phi(\omega_0^*)\sum_{\ell:\, y_\tau\in\ell}\Bigg[\frac{\partial v(q_A^e,\theta\tilde{q}_{A0}^e)}{\partial q_{A\ell}^e}+\frac{\partial v(q_B^e,\theta\tilde{q}_{B0}^e)}{\partial q_{B\ell}^e} \\ +\theta\left(\frac{\partial v(q_A^e,\theta\tilde{q}_{A0}^e)}{\partial\tilde{q}_{A0}^e}+\frac{\partial v(q_B^e,\theta\tilde{q}_{B0}^e)}{\partial\tilde{q}_{B0}^e}\right)h_{0\ell}\Bigg]f(\boldsymbol{\alpha})=\frac{\partial\kappa(\mathbf{c}_A^*)}{\partial c_{Ay_r}}, \quad (23)$$

where

$$f(\boldsymbol{\alpha})=\begin{cases}\frac{\alpha_{y_\tau}}{\sum_{y_s\in\ell}\alpha_{y_s}} & \text{if } q_{A\ell}^e(\mathbf{c}^*)=\left(1+\frac{\sum_{y_s\in\ell}\alpha_{y_s}(c_{jy_s}-c_{-jy_s})}{\sum_{y_s}\alpha_{y_s}}\right)q_{j\ell}^e\\ 0 & \text{otherwise.}\end{cases}$$

For statement (i), as $\theta\to0$, the FOC becomes

$$p_A(\mathbf{q}^e(\mathbf{c}^*))\phi(\omega_0^*)\sum_{\ell:\, y_\tau\in\ell}\left(\frac{\partial v(q_A^e,0)}{\partial q_{A\ell}^e}+\frac{\partial v(q_B^e,0)}{\partial q_{B\ell}^e}\right)f(\boldsymbol{\alpha})=\frac{\partial\kappa(\mathbf{c}_A^*)}{\partial c_{Ay_r}}.$$

At $\mathbf{c}=0$,

$$p_A(\mathbf{q}^e(0))\phi(\omega_0^*)\sum_{\ell:\, y_\tau\in\ell}\left(\frac{\partial v(q_A^e,0)}{\partial q_{A\ell}^e}+\frac{\partial v(q_B^e,0)}{\partial q_{B\ell}^e}\right)f(\boldsymbol{\alpha})>0$$

for at least one y_r. By Assumption 2(iv), $\kappa(\mathbf{0})\to0$. By continuity, this holds for θ sufficiently small. Define $\underline{\theta}(\mathbf{q}^e)$ as the largest θ such that the inequality holds.

Statement (ii) follows from $p_A(\mathbf{q}^e(\mathbf{c}^*))\phi(\omega_0^*)>0$.

For statement (iii), consider all ℓ such that $\{\ell:\, y_r\in\ell\}=\{\ell:\, y_s\in\ell\}$. The corresponding FOCs are

$$p_A(\mathbf{q}^e(\mathbf{c}^*))\phi(\omega_0^*)\sum_{\ell:\, y_\tau\in\ell}\Bigg[\frac{\partial v(q_A^e,\theta\tilde{q}_{A0}^e)}{\partial q_{A\ell}^e}+\frac{\partial v(q_B^e,\theta\tilde{q}_{B0}^e)}{\partial q_{B\ell}^e} \\ +\theta\left(\frac{\partial v(q_A^e,\theta\tilde{q}_{A0}^e)}{\partial\tilde{q}_{A0}^e}+\frac{\partial v(q_B^e,\theta\tilde{q}_{B0}^e)}{\partial\tilde{q}_{B0}^e}\right)h_{0\ell}\Bigg]f(\boldsymbol{\alpha})=\frac{\partial\kappa(\mathbf{c}_A^*)}{\partial c_{Ay_r}},$$

$$p_A(\mathbf{q}^e(\mathbf{c}^*))\phi(\omega_0^*)\sum_{\ell:\, y_\tau\in\ell}\Bigg[\frac{\partial v(q_A^e,\theta\tilde{q}_{A0}^e)}{\partial q_{A\ell}^e}+\frac{\partial v(q_B^e,\theta\tilde{q}_{B0}^e)}{\partial q_{B\ell}^e} \\ +\theta\left(\frac{\partial v(q_A^e,\theta\tilde{q}_{A0}^e)}{\partial\tilde{q}_{A0}^e}+\frac{\partial v(q_B^e,\theta\tilde{q}_{B0}^e)}{\partial\tilde{q}_{B0}^e}\right)h_{0\ell}\Bigg]f(\boldsymbol{\alpha})=\frac{\partial\kappa(\mathbf{c}_A^*)}{\partial c_{Ay_s}}.$$

The left-hand side of each FOC is identical except for $f(\boldsymbol{\alpha})$ Thus, the left-hand side of the FOC for c_{Ay_r} is greater than (less than) [equal to] the left-hand side of the FOC for c_{Ay_s} if and only if

$$\frac{\alpha_{y_r}}{\sum_{y_t\in\ell}\alpha_{y_t}} > \ (<)\ [=]\frac{\alpha_{y_s}}{\sum_{y_t\in\ell}\alpha_{y_t}}$$
$$\alpha_{y_r} > \ (<)\ [=]\alpha_{y_s}.$$

An analogous argument holds for platform B. □

Proof of Proposition 7.

Proof. Without utilizing Lemma 2, the FOC with respect to c_{Ay_r} is given by

$$p_A(\mathbf{q}^e(\mathbf{c}^*)) \sum_{m=1}^{L} \phi(\omega_m^*) \sum_{\ell:\, y_s \in \ell} \left[\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial q_{Am}^e} + \frac{\partial v(q_B^e, \theta \tilde{q}_{Bm}^e)}{\partial q_{Bm}^e} \right.$$
$$\left. + \theta \left(\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial \tilde{q}_{Am}^e} + \frac{\partial v(q_B^e, \theta \tilde{q}_{Bm}^e)}{\partial \tilde{q}_{Bm}^e} \right) h_{m\ell} \right] \frac{\alpha_{y_s}}{\sum_{y_t \in \ell} \alpha_{y_t}} \leq \frac{\partial \kappa(\mathbf{c}_A^*)}{\partial c_{Ay_r}}.$$

At $\mathbf{c} = 0$, $\omega_m^* = \omega_\ell^*$ for all m, ℓ. It is sufficient to show that the left-hand side of the above is nonpositive at $\mathbf{c} = 0$:

$$\sum_{m=1}^{L} \sum_{\ell:\, y_s \in \ell} \left[\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial q_{Am}^e} + \frac{\partial v(q_B^e, \theta \tilde{q}_{Bm}^e)}{\partial q_{Bm}^e} + \theta \left(\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial \tilde{q}_{Am}^e} + \frac{\partial v(q_B^e, \theta \tilde{q}_{Bm}^e)}{\partial \tilde{q}_{Bm}^e} \right) h_{m\ell} \right] \frac{\alpha_{y_s}}{\sum_{y_t \in \ell} \alpha_{y_t}} \leq 0.$$

For θ sufficiently large, it is sufficient that

$$\sum_{m=1}^{L} \sum_{\ell:\, y_s \in \ell} \left[\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial \tilde{q}_{Am}^e} h_{m\ell} \right] = \sum_{\ell:\, y_s \in \ell} \sum_{m=1}^{L} \left[\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial \tilde{q}_{Am}^e} h_{m\ell} \right] \leq 0$$

Note that there exists a $\varepsilon > 0$ such that if $\sum_{m \neq \ell} |h_{m\ell}| - h_{\ell\ell} > \varepsilon$, then for each ℓ,

$$\sum_{m=1}^{L} \left[\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial \tilde{q}_{Am}^e} h_{m\ell} \right] \leq 0.$$

As this holds for each ℓ, it holds for all $\ell : y \in \ell$, proving the first statement.

To prove (i) and (ii), assume sufficiently strong in-group bias. Reversing the inequalities, an identical argument as above follows, except if $\sum_{m \neq \ell} |h_{m\ell}| - h_{\ell\ell} < -\varepsilon$, then

$$\sum_{m=1}^{L} \left[\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial \tilde{q}_{Am}^e} h_{m\ell} \right] > 0,$$

completing the proof. □

Proof of Proposition 8.

Proof. This proof follows a nearly identical argument to that of Proposition 7. The FOC with respect to c_{Ay_r} is again given by (23). At $\mathbf{c} = 0$, the left-hand side must be nonpositive. Assuming θ sufficiently large, (23) simplifies to

$$\sum_{m=1}^{L} \phi(\omega_m^*) \sum_{\ell:\, y_s \in \ell} \left(\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial \tilde{q}_{Am}^e} + \frac{\partial v(q_B^e, \theta \tilde{q}_{Bm}^e)}{\partial \tilde{q}_{Bm}^e} \right) h_{m\ell} \leq 0.$$

Define

$$A_m = \phi(\omega_m^*) \left(\frac{\partial v(q_A^e, \theta \tilde{q}_{Am}^e)}{\partial \tilde{q}_{Am}^e} + \frac{\partial v(q_B^e, \theta \tilde{q}_{Bm}^e)}{\partial \tilde{q}_{Bm}^e} \right),$$

so the above can be rewritten as $\sum_{m=1}^{L} \sum_{\ell:\, y_s \in \ell} A_m h_{m\ell}$. The argument then proceeds identically to that of Proposition 7. $\square$

Proof of Proposition 9.

Proof. Without loss of generality, suppose that $|h_{11}| > \cdots > |h_{LL}|$ and $h_{m\ell} = h'$ for all $m \neq \ell$. Recall from Proposition 4 that

$$\frac{d\pi_A(\mathbf{p}(\mathbf{q}^e))}{dq_{A\ell}^e} = np_A(\mathbf{q}^e)\phi(\omega_0^*) \left[\left(\frac{\partial v(q_A^e, \theta \tilde{q}_{A0}^e)}{\partial q_{A\ell}^e} + \frac{\partial v(q_B^e, \theta \tilde{q}_{B0}^e)}{\partial q_{B\ell}^e} \right) + \theta \left(\frac{\partial v(q_A^e, \theta \tilde{q}_{A0}^e)}{\partial \tilde{q}_{A0}^e} + \frac{\partial v(q_B^e, \theta \tilde{q}_{B0}^e)}{\partial \tilde{q}_{B0}^e} \right) h_{0\ell} \right].$$

For θ sufficiently large and sufficiently strong out-group bias ($\sum_{m \neq \ell} h_{\ell m} - |h_{\ell\ell}| < -\varepsilon$ for some positive ε), profits are decreasing in $q_{A\ell}^e$ if $\sum_{m=1}^{L} h_{m\ell} < 0$.

By definition 5, $h_{m\ell} > 0$ for all $m \neq \ell$ and $h_{\ell\ell} < 0$ for all ℓ. Aggregating over all ℓ, profits are maximized by minimizing $\sum_\ell q_{A\ell} h_{\ell\ell}$. With the $h_{\ell\ell}$ fixed, for every q_A^e, profits are maximized when $q_{A1} < q_{A2} < \cdots < q_{AL}$. An analogous argument holds for platform B. $\square$

Proof of Proposition 10.

Proof. Suppose that $K = 2$, $\bar{Y} = L$, $\mathbf{q}^e = \mathbb{E}\mathbf{Q}^e$, and that θ is sufficiently large. As $\bar{Y} = L$, I use traits and types interchangeably. As θ is sufficiently large, size effects can be ignored and composition effects dominate. By Lemma 3, at least one SPE exists and is given by the solution to the system of $2L$ equations.

Define $\bar{h} = \max_{\ell,m} h_{\ell m}$ and $\underline{h} = \min_{\ell,m} h_{\ell m}$.

For (i), suppose that $\bar{h} - \underline{h} < \varepsilon$ for a small positive ε. Consider the cultivation profile $\mathbf{c}^*$ such that $c_{A\ell}^* > 0$ and $c_{B\ell}^* = 0$ for all $\ell \in L_1$, while $C_{A\ell}^* = 0$ and $c_{B\ell}^* > 0$ for all $\ell \in L_2$. Under $\mathbf{c}^*$, $q_A^e = n/2$ and $q_{A\ell}^e > n_\ell/2$ for all $\ell \in L_1$ and $q_{A\ell}^e < n_\ell/2$ for all $\ell \in L_2$. Now, consider a unilateral deviation by platform A to a profile $\mathbf{c}'$, where all values are identical to $\mathbf{c}^*$ except that $c_{A\ell} = \varepsilon' > 0$ for some $\ell \in L_2$. By Assumption 2, if this deviation does not exist, there can be no deviation to a value greater than ε'. The introduction of a user from L_2 imposes a negative externality on all users from L_1. By Proposition 4, $\frac{d\pi_A(\mathbf{p}(\mathbf{q}^e(\mathbf{c}')))}{dq^e_{A\ell}} < 0$ as $h_{m\ell} < 0$ for all $m \in L_1$ and $q^e_{Am} > q^e_{A\ell}$ for each $m \in L_1$ and $\ell \in L_2$. The composition worsens for a strict majority of users and improves for a small minority. As $\bar{h} - \underline{h} < \varepsilon$, the net effect is negative. Thus, there is no profitable deviation.

Now consider a unilateral deviation to a profile $\mathbf{c}''$, where all is identical to $\mathbf{c}^*$ except that $c_{A\ell} = 0$ for some $\ell \in L_1$. Then, $q^e_{A\ell}(\mathbf{c}'') = n_\ell/2 < \mathbf{q}^e(\mathbf{c}^*)$. Moreover, $\tilde{q}^e_{A\ell}(\mathbf{c}'') < \tilde{q}^e_{A\ell}(\mathbf{c}^*)$ for all $\ell \in L_1$. Given Assumption 2(iv), $\mathbf{c}''$ yields a strict decrease in platform A's profits via a lower price due to poorer composition and fewer users due to the loss of a cultivated trait. Hence, no such deviation exists. By symmetry, a similar argument holds for platform B.

To prove uniqueness, consider any profile $\mathbf{c} \neq \mathbf{c}^*$. Cases in which a platform targets a subset of a partition are ruled out by the above. The only remaining candidate $\mathbf{c}$ is both platforms targeting the same partition. However, if one platform deviates by targeting the alternate partition, it receives a strict increase in profits through increased composition. The mathematical argument is analogous to the one presented above.

For (ii), suppose group 1 is dominant (by Definition 6). Consider the profile $\mathbf{c}^*$ with $c^*_{j\ell} > 0$ for all j and all $\ell \in L_1$. By symmetry, $c^*_{A\ell} = c^*_{B\ell}$ for all ℓ, so $q^e_A = q^e_B = n/2$ and $q^e_{A\ell} = q^e_{B\ell} = n_\ell/2$ for all ℓ. A deviation by platform A to $\mathbf{c}'$ in which all is identical except $c_{A\ell} < c_{B\ell}$ for some $\ell \in L_1$. As $q^e_A(\mathbf{c}') < q^e_A(\mathbf{c}^*)$, there is a negative size effect. For all type z_m users, $m \in L_1$, $\tilde{q}^e_{Am}(\mathbf{c}') < \tilde{q}^e_{Am}(\mathbf{c}^*)$. Moreover, for each $m \notin L_1$, the net change $\tilde{q}^e_{Am}(\mathbf{c}') - \tilde{q}^e_{Am}(\mathbf{c}^*)$ implies (by Assumption 3) that $p_A(\mathbf{q}^e(\mathbf{c}')) < p_A(\mathbf{q}^e(\mathbf{c}^*))$. By group 1's dominance, this implies a strict decrease in profits.

The only alternative deviation necessary to consider is platform A cultivating a type z_m individual with $m \in L_2$. Again, by Definition 6, the negative externality imposed on group 1 dominates the positive externality on group 2, implying a strict decrease in profits. Thus, there is no unilateral deviation. □

Proof of Proposition 11.

Proof. As in the text, if there is a single platform A, then a constant ξ_A and $\theta = 0$ implies that profits are maximized at $p^*(0) = \xi_A + v(q^e_A, 0)$ where $p^*(0)$ is $p^*(\theta)$ evaluated at $\theta = 0$. In equilibrium, $q^e_A = n$. Profits are np^*. Under J platforms, each platform j maximizes profits at price

$p_j^*(0) = \xi_A + v(q_j^e, 0)$. If $q_j^e < n$, then $p^*(0) > p_j^*(0)$ for all j. Total profits across the J platforms are $\sum_j q_j^e p_j^*(0) < \sum_j q_j^e p^*(0) = np^*(0)$. By continuity, the inequality holds for small but positive θ. $\square$

Proof of Proposition 12.

Proof. Suppose that H is sufficiently large and H consists of either in-group bias, out-group bias, or grouping. Define $\underline{\ell}$ as the type z_ℓ user with the lowest heterogeneity-weighted installed base value in platform A.

A single platform A maximizes prices by setting

$$p^*(\theta) = \begin{cases} \xi_A + v(n, \theta \tilde{q}^e_{A\underline{\ell}}) & \text{if } q_A^e = n \\ \xi_A + v(q_A^e, \theta \tilde{q}^e_{A\underline{\ell}}) & \text{if } q_A^e < n. \end{cases}$$

If $q_A^e < n$, then creating a second platform with the remaining $n - q_A^e$ users completes the proof. If $q_A^e = n$, then by the construction of H, there must exist at least one type z_ℓ such that by removing the type z_ℓ, $p^*(\theta)$ increases. Then by the above case, creating at least one new platform for the removed type(s) completes the proof. $\square$

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