Supplemental Materials: Minimum Resale Price Maintenance Can Reduce Prices

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This supplementary appendix shows that linear and constant elasticity demands satisfy $\underline{w} = w^*$ and Conditions 1 and 2 of Proposition 1. It also shows Condition 1 is satisfied by logit demand and Condition 2 is satisfied for a wide range of parameters for logit demand, thus ensuring $\underline{p}^*(\underline{w}) < p^m(w^*)$.

S1 Baseline with linear demand

Let market demand be given by $F(p) = a - p$, and for simplicity, the upstream manufacturer has marginal cost $c = 0$. Since we assume $\delta > \frac{n-1}{n}$, retailers collude at the monopoly price and set $p = p^m = \frac{a+w}{2}$. The upstream manufacturer maximizes $\pi_u = w \times F(p) = w \times (a - p)$. Substituting $p = \frac{a+w}{2}$, and solving, we find $w^* = \frac{a}{2}$.

S1.1 Minimum RPM with linear demand

Now assume as in Section 2.1, the manufacturer sets w and $\underline{p}$ and that absent minimum RPM, retailer collusion at $p = p^m$ will occur.

By Lemma 2, the manufacturer will set $\underline{p}$ such that $\delta = \delta_{\underline{p}}$, which implies $(\underline{p}(w) - w)F(\underline{p}(w)) = \left(n - \frac{n-1}{\delta}\right)(p^m(w) - w)F(p^m(w))$. Substituting the terms derived from the linear demand above, we obtain

$$(\underline{p} - w)(a - \underline{p}) = \left(n - \frac{n-1}{\delta}\right)\left(\frac{a+w}{2} - w\right)\left(a - \frac{a+w}{2}\right).$$

Solving for $\underline{p}$, we get

$$\begin{aligned}\underline{p}^* &= \frac{\delta(a+w) - \sqrt{\delta(1-\delta)(n-1)(a-w)^2}}{2\delta} \\ &= \frac{a+w}{2} - \frac{(a-w)}{2}\underbrace{\sqrt{\frac{(1-\delta)(n-1)}{\delta}}}_{\equiv \kappa}. \qquad \text{(S.1)}\end{aligned}$$

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By Lemma 2 the market price will now be $p = \underline{p}$, so the manufacturer maximizes $\pi_u = w \times F(p) = w(a-p) = w(a - \underline{p}^*)$ with respect to w. Substituting $\underline{p}^*$ from (S.1),

$$\begin{aligned}\pi_u &= (w-c)\left(a - \frac{a+w}{2} + \frac{a-w}{2}\kappa\right)\\ &= (w-c)\left(\frac{a-w}{2}\right)(1+\kappa)\\ &= \frac{1+\kappa}{2}(w-c)(a-w).\end{aligned}$$

Solving the first order condition with respect to w, we find $\underline{w} = \frac{a}{2}$, thus $\underline{w} = w^*$. Note as well that $\underline{p}^*$ is Lipschitz continuous with respect to w, and since $\underline{w} = w^*$, linear demand satisfies Conditions 1 and 2 of Proposition 1.

S2 Baseline with constant elasticity demand

Let market demand be given by $F(p) = ap^{\varepsilon}$, where the elasticity of demand $\varepsilon < -1$. The upstream manufacturer has constant marginal cost c. Since we assume $\delta > \frac{n-1}{n}$, retailers collude at the monopoly price, $p^m = \arg\max_p (p-w)ap^{\varepsilon}$, thus, $p^m = (\frac{\varepsilon}{\varepsilon+1})w$. The upstream manufacturer maximizes $\pi_u = (w-c) \times F(p) = (w-c) \times ap^{\varepsilon}$. Substituting $p = (\frac{\varepsilon}{\varepsilon+1})w$, and solving, we find $w^* = (\frac{\varepsilon}{\varepsilon+1})c$.

S2.1 Minimum RPM with constant elasticity demand

Now assume as in Section 2.1, the manufacturer sets w and $\underline{p}$ and that absent minimum RPM, retailer collusion at $p = p^m$ will occur.

By Lemma 2, the manufacturer will set $\underline{p}$ such that $\delta = \delta_{\underline{p}}$, which implies $(\underline{p}(w) - w)F(\underline{p}(w)) = \left(n - \frac{n-1}{\delta}\right)(p^m(w) - w)F(p^m(w))$. Substituting the terms derived from the constant elasticity demand above, we obtain

$$\begin{aligned}(\underline{p} - w)a\underline{p}^{\varepsilon} &= \left(n - \frac{n-1}{\delta}\right)\left(\left(\frac{\varepsilon}{\varepsilon+1}\right)w - w\right)a\left(\left(\frac{\varepsilon}{\varepsilon+1}\right)w\right)^{\varepsilon}\\ &= \underbrace{\left(n - \frac{n-1}{\delta}\right)\left(\left(\frac{\varepsilon}{\varepsilon+1}\right) - 1\right)\left(\frac{\varepsilon}{\varepsilon+1}\right)^{\varepsilon}}_{\equiv\kappa} a w^{\varepsilon+1}.\end{aligned}$$

This implies

$$(\underline{p} - w)\underline{p}^{\varepsilon} = \kappa w^{\varepsilon+1}. \tag{S.2}$$

Note that κ is a constant determined by the model primitives, and both sides of (S.2) are homogeneous of degree $\varepsilon + 1$ in $\underline{p}$ and w. Defining the left hand side of S.2 to be $g(\underline{p}, w) = (\underline{p} - w)\underline{p}^{\varepsilon}$, this implies that $g(\lambda\underline{p}, \lambda w) = \lambda^{\varepsilon+1} g(\underline{p}, w)$. Substitute $\lambda = \frac{1}{w}$, and define $m = \frac{\underline{p}}{w}$. This implies

$$\begin{aligned}g(m,1) &= \left(\frac{1}{w}\right)^{\varepsilon+1} g(\underline{p}, w)\\ w^{\varepsilon+1} g(m,1) &= g(\underline{p}, w).\end{aligned}$$

Substituting into S.2, we obtain

$$\begin{aligned} w^{\varepsilon+1}(m-1)m^{\varepsilon} &= \kappa w^{\varepsilon+1} \\ (m-1)m^{\varepsilon} &= \kappa. \end{aligned} \tag{S.3}$$

Thus, since κ is a constant, for constant elasticity demand, the optimal minimum resale price is a constant mark up over w: $\underline{p}^* = m \times w$, where $m < \frac{\varepsilon}{\varepsilon+1}$ by Lemma 2. Given this, the upstream manufacturer solves $\underline{w} = \arg\max_w \pi_u = (w-c) \times F(\underline{p}) = \arg\max_w (w-c) \times a(mw)^{\varepsilon}$, thus $\underline{w} = w^* = c(\frac{\varepsilon}{\varepsilon+1})$. Note as well, that $\underline{p}^* = m \times w$ is Lipschitz continuous with respect to w with $\bar{b} = m$, and since $\underline{w} = w^*$, constant elasticity demand satisfies Conditions 1 and 2 of Proposition 1.

S3 Logit Demand

Suppose that the logit demand is given by $F(p) = \frac{1}{1+e^{\alpha+\beta p}}$ for $\alpha \in \mathbb{R}$ and $\beta > 0$. Since we cannot analytically compute the wholesale and retail prices, we first show that this demand satisfies Condition 1 of Proposition 1: $\underline{p}^*(w)$ satisfies $|\underline{p}^*(w') - \underline{p}^*(w)| \leq \bar{b}|w'-w|$ for all w and w' and $\bar{b} = 1$. Then, we numerically illustrate for a range of parameters that condition 2 can be satisfied.

S3.1 Condition 1 of Proposition 1

To satisfy Condition 1 for $\bar{b} = 1$, it is sufficient that $\frac{d\underline{p}^*}{dw} \in [0,1]$. First, note that logit demand is log-concave, implying an increasing hazard rate and therefore a unique maximizer of $(p-w)F(p)$, $p^m(w)$, for every w. Next, recall from Lemma 2 that $\underline{p}^*$ is given by the smallest $\underline{p}$ that satisfies

$$(\underline{p}-w)F(\underline{p}) = \underbrace{\left(n - \frac{n-1}{\delta}\right)}_{\equiv\theta} \underbrace{(p^m(w)-w)F(p^m(w))}_{\equiv\pi^m(w)}. \tag{S.4}$$

Next, note that under logit demand,

$$\frac{\partial}{\partial p}(p-w)F(p) = F(p)\left(1 - \beta(p-w)(1-F(p))\right). \tag{S.5}$$

Evaluating (S.5) at the monopoly price (so the above equals zero) yields

$$(p^m(w)-w)(1-F(p^m(w))) = \frac{1}{\beta}.$$

Let $g(w) = \underline{p}^*(w) - w$ denote the markup, so (S.4) can be rewritten as

$$g(w)F(g(w)+w) = \theta\pi^m(w). \tag{S.6}$$

Differentiating both sides of (S.6) with respect to w yields

$$\frac{dg(w)}{dw}\left(F(g(w)+w) + g(w)\frac{dF(g(w)+w)}{dp}\right) + g(w)\frac{dF(g(w)+w)}{dp} = -\theta F(p^m(w)), \tag{S.7}$$

where the right-hand side follows from the envelope theorem. Solving (S.7) for $\frac{dg(w)}{dw}$ yields

$$\frac{dg(w)}{dw} = \frac{-\theta F(p^m(w)) - g(w)\frac{dF(g(w)+w)}{dp}}{F(g(w)+w) + g(w)\frac{dF(g(w)+w)}{dp}} \tag{S.8}$$

Note that the denominator is equal to $\frac{d}{dp}(p-w)F(p)$ evaluated at $p = \underline{p}^*(w)$. As $\underline{p}^*(w) \in [w, p^m(w))$ (by Lemmas 1 and 2), $F(g(w)+w) + g(w)\frac{dF(g(w)+w)}{dp}$ is strictly positive. Next, we show that the numerator is nonpositive.

Recall that for the above logit demand specification, $\frac{dF(p)}{dp} = -\beta F(p)(1-F(p))$, so we can rewrite the numerator of (S.8) as

$$-\theta F(p^m(w)) + g(w)\beta F(\underline{p}^*(w))(1 - F(\underline{p}^*(w))).$$

Using (S.6), we can rewrite the above expression as

$$\theta F(p^m(w))\left(\beta(p^m(w)-w)(1-F(\underline{p}^*(w))) - 1\right). \tag{S.9}$$

As $(p^m(w)-w)(1-F(p^m(w))) = \frac{1}{\beta}$ and $\underline{p}^*(w) < p^m(w)$, it follows that $(p^m(w)-w)(1-F(\underline{p}^*(w))) < \frac{1}{\beta}$, so (S.9) is less than zero, verifying that $\frac{dg(w)}{dw} \leq 0$. Since $\underline{p}^* = g(w) + w$,

$$\frac{d\underline{p}^*(w)}{dw} = \frac{dg(w)}{dw} + 1.$$

Because $\frac{dg(w)}{dw} \leq 0$, it follows that $\frac{d\underline{p}^*}{dw} \leq 1$.

Lastly, we show that $\frac{d\underline{p}^*}{dw} > 0$. Invoking the implicit function theorem on (S.4) yields

$$\frac{d\underline{p}^*}{dw} = \frac{F(\underline{p}^*(w)) - \theta F(p^m(w))}{F(\underline{p}^*(w)) + (\underline{p}^*(w) - w)\frac{dF(\underline{p}^*(w))}{dp}}.$$

The denominator is strictly positive since $\underline{p}^* < p^m$ and $F(\cdot)$ is decreasing, and since $\underline{p}^*(w) < p^m(w)$ and $\theta \in (0,1)$ for $\delta > \frac{n-1}{n}$, the numerator is as well. Hence, $\frac{d\underline{p}^*}{dw} \in [0,1]$.

S3.2 Condition 2 of Proposition 1

To illustrate that Proposition 1 can hold under logit demand $F(p) = \frac{1}{1+e^{\alpha+\beta p}}$, we conduct a numerical analysis across a wide range of parameter values. Table 1 summarizes the parameters tested.

Across the 900 specifications, all of them satisfied Condition 2. Moreover, in all specifications, $\underline{w} \geq w^*$, strictly so for the majority. Thus, with minimum RPM, in lowering the retail price from the downstream monopoly level $p^m(w^*)$ to $\underline{p}^*(w^*)$, the supplier has the incentive to increase w to increase its markup from $w^* - c$ to $\underline{w} - c$, which pushes $p^m(w^*)$ to $p^m(\underline{w})$ and subsequently $\underline{p}^*(w^*)$ up to $\underline{p}^*(\underline{w})$. However, in all 900 specifications, $\underline{p}^*(\underline{w}) < p^m(w^*)$, so minimum RPM reduces the retail price. Figure 1 plots all 900 specifications across two dimensions: the value of $|\underline{w} - w^*|$ and the values of $p^m(w^*) - \underline{p}^*(w^*)$. Every point above the 45-degree line corresponds to a specification that satisfies Condition 2 of Proposition 1 for $\bar{b} = 1$.

For concreteness, Table 2 highlights nine specific cases. In each presented specification, $c = 0$, $\delta = 0.75$, and $n = 2$.

Parameter	Values
α	$\{-6, -4, -2, -1, 0, 1, 2, 4, 6\}$
β	$\{0.2, 0.4, 0.6, 0.8, 1, 1.2, 1.4, 1.6, 1.8, 2\}$
δ	$\{0.67, 0.75, 0.9, 0.99, 0.999\}$
n	$\{2, 3\}$
c	$\{0\}$

Table 1: Summary of parameter values across all specifications.

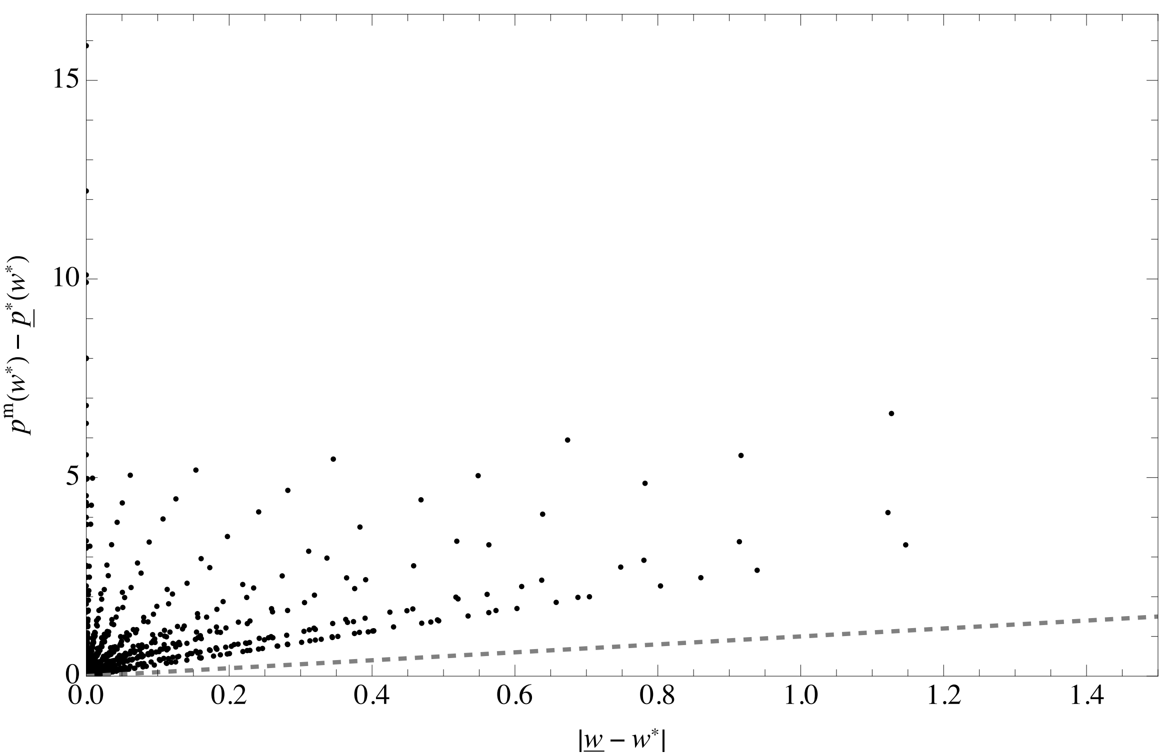

Figure 1: Numerical Illustration of Condition 2 of Proposition 1 for logit demand.

Parameter Values		w^*	$\underline{w}$	$p^m(w^*)$	$\underline{p}^*(\underline{w})$
$\alpha = -4$	$\beta = 0.6$	4.59	5.02	7.36	6.09
$\alpha = 0$	$\beta = 0.6$	2.01	2.08	3.85	2.74
$\alpha = 4$	$\beta = 0.6$	1.67	1.68	3.35	2.26
$\alpha = -4$	$\beta = 1$	2.76	3.01	4.42	3.65
$\alpha = 0$	$\beta = 1$	1.21	1.24	2.31	1.64
$\alpha = 4$	$\beta = 1$	1.00	1.01	2.01	1.35
$\alpha = -4$	$\beta = 1.4$	1.97	2.15	3.15	2.61
$\alpha = 0$	$\beta = 1.4$	0.86	0.89	1.65	1.17
$\alpha = 4$	$\beta = 1.4$	0.72	0.72†	1.43	0.97

† When rounding to four decimal places for $\alpha = 4$ and $\beta = 1.4$, $w^* = 0.7178$ and $\underline{w} = 0.7185$, so $\underline{w} > w^*$.

Table 2: Sample of logit specifications. All values are rounded to two decimal places.